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Development of Analytical Methods Applicable to Test Data

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In a variety of situations, an undesired shock excitation is applied to a master structure that supports shock-sensitive equipment. Often, one wishes to design and test a master structure that transmits the least amount of shock energy to the attached equipment. In scaled testing of new designs, a major task is to design and construct "equipment emulators" – inexpensive mechanical systems which approximately mimic the dynamic behavior of the actual full-scale equipment as seen by the master structure. In this report, the fundamental issues relating to equipment emulation are identified and design guidelines are proposed based on current NAVSEA vibration and shock qualification standards. Furthermore, a new method is presented for assessing the fidelity of equipment emulators and for interpreting test data taken in the presence of imperfect emulators. The proposed approach uses easily obtainable frequency-domain impedance descriptions of the master structure and actual equipment at the attachment points. The ideas are illustrated by application to the emulation of commercial-grade electronic cabinets.

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Development of Analytical Methods Applicable to Test Data

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Prepared for:
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Development of Analytical Methods Applicable to Test Data

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Equipment Emulators for UNDEX Testing

Pierre Dupont

Aerospace and Mechanical Engineering

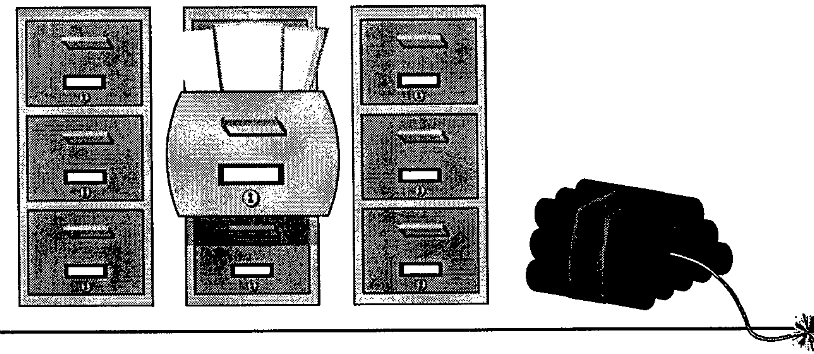
Boston University

9 March, 1998

Out of the Tower and into the Laboratory...

- *What goes on in the "real" world?*
- *What does it take to float a boat?*
- Collaborators:
  - *Bill Martin* — *NSWCCD*
  - *Liming Salvino, Ph.D.* — *NSWCCD*
  - *Charles Milligan, Ph.* — *NSWCCD*
  - *Jeff O'Brien* — *ETC*

Talk Outline



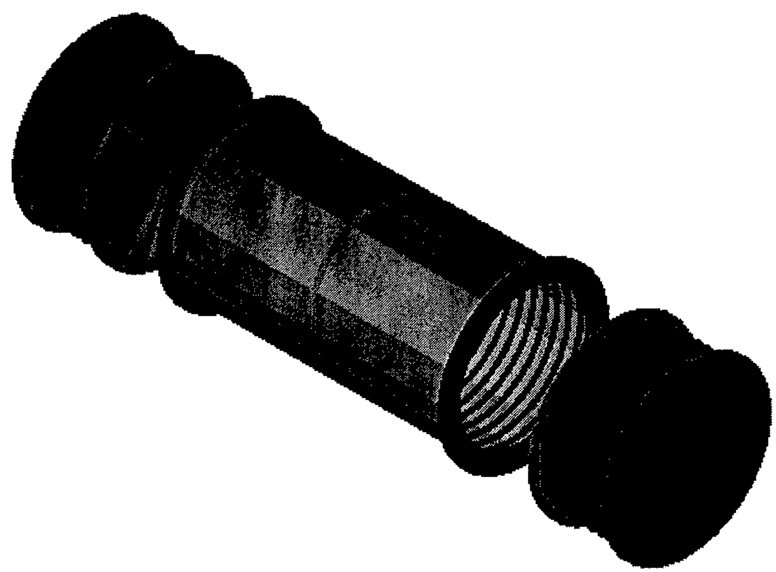
- Truss and Mount Design
- Shock Acceptance Tests
- Cabinet Modeling

Truss and Mount Design:

- ***Problem Statement:*** *Formulate design guidelines for truss and mounts so that COTS equipment and people survive.*
- ***Constraints:***
  - *Preserve acoustic performance.*
  - *Rattle space is specified.*

1/4 Scale Testing

- Goals:
  - *Predict truss response.*
  - *Predict response of equipment in cabinet.*



Equipment Emulators

- Benefits of using appropriate emulators:
  - Test realism
  - Cost savings
- Two questions to address:

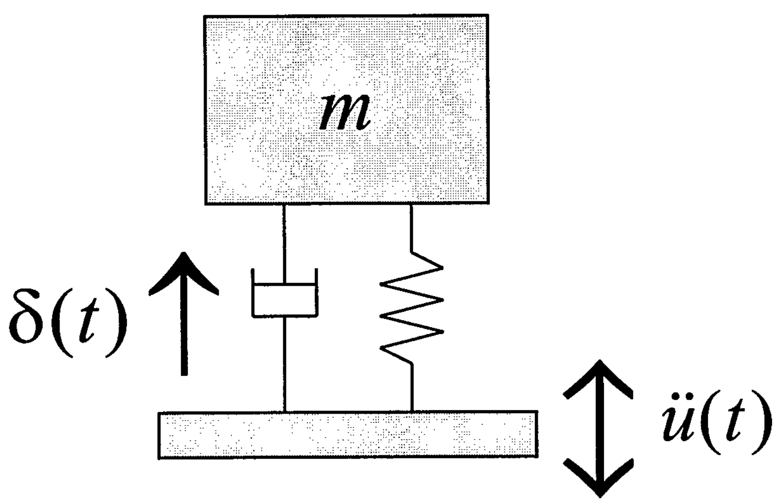
  (1) What are the critical load-point cabinet dynamics for accurate prediction of truss response?

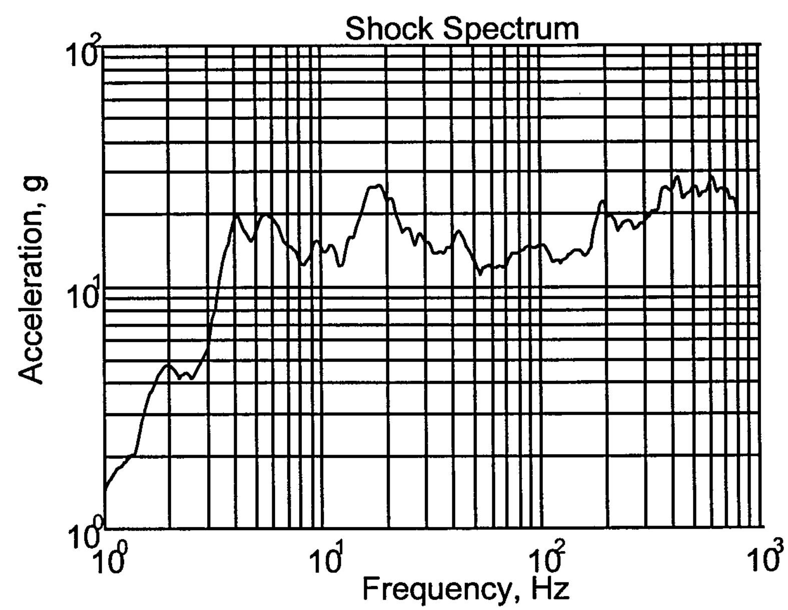
  (2) How does equipment response vary with location in a cabinet?

Shock Spectrum

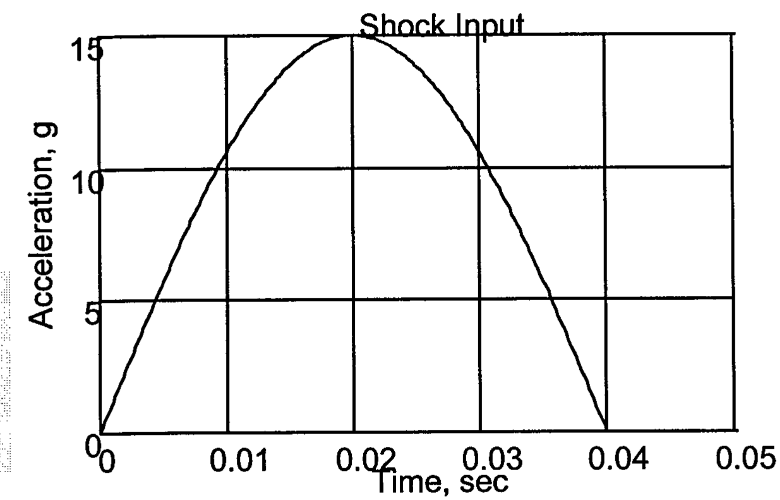
$$\delta(t) = \frac{1}{\omega_d}\int_0^t \ddot{u}(\tau)e^{-\zeta\omega_n(t-\tau)}\sin\omega_d(t-\tau)\,d\tau$$

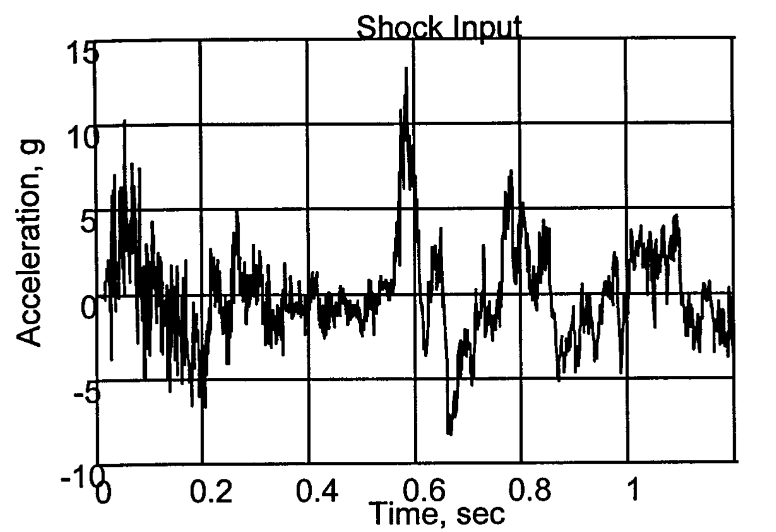
$$A(\omega_n,\zeta) = \frac{\omega_n^2}{g}\delta_{\max}(\omega_n,\zeta)$$

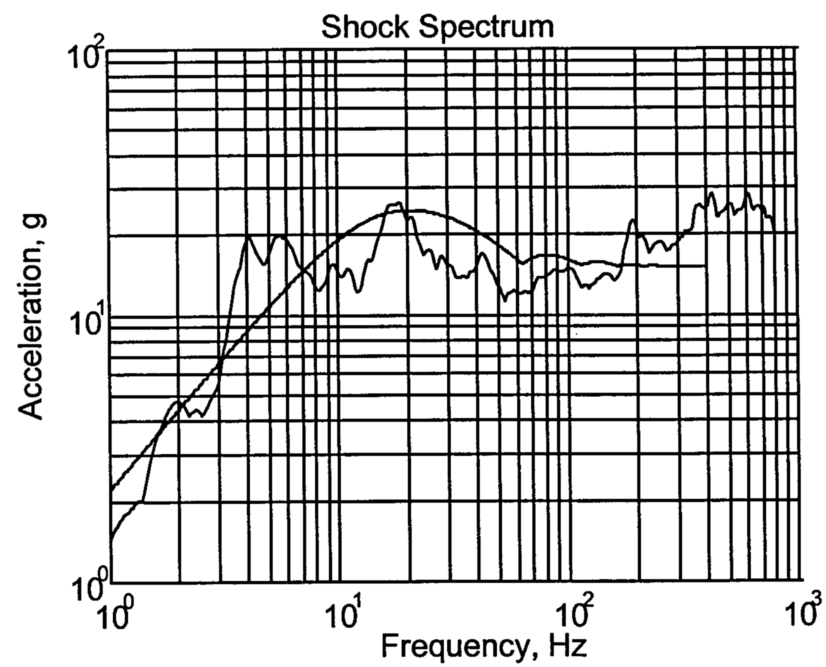




Comparing Shock Spectra

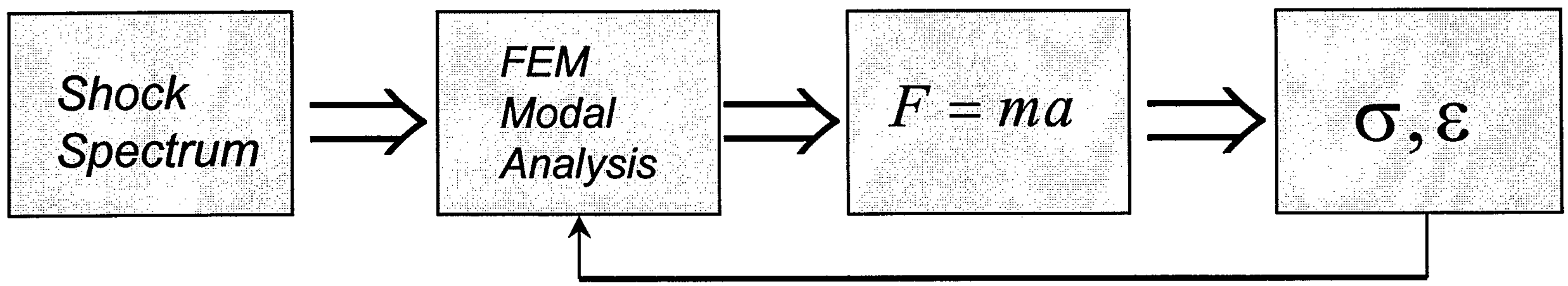




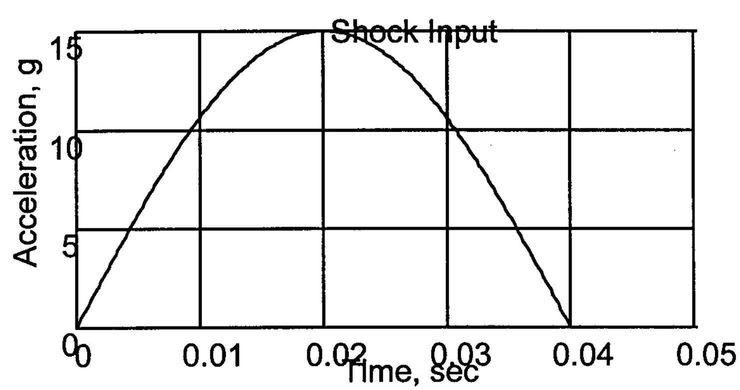


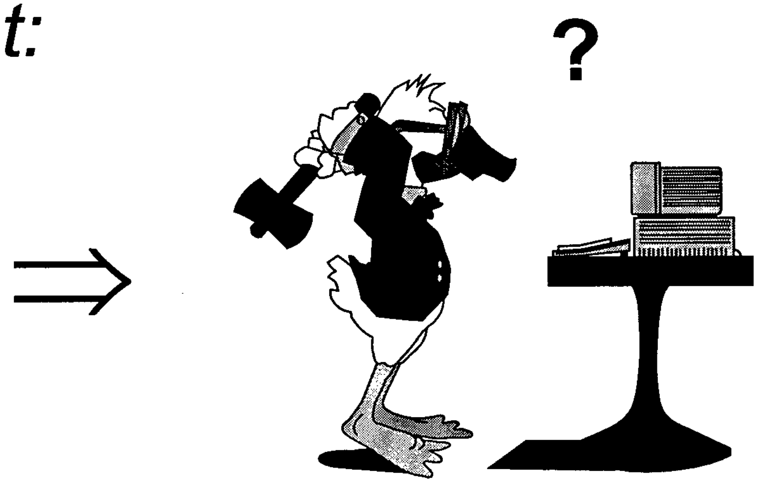
Designing with Shock Spectra

- **Structural Design:**



- **Electronic Equipment:**





Relating shock spectrum to stress and strain:

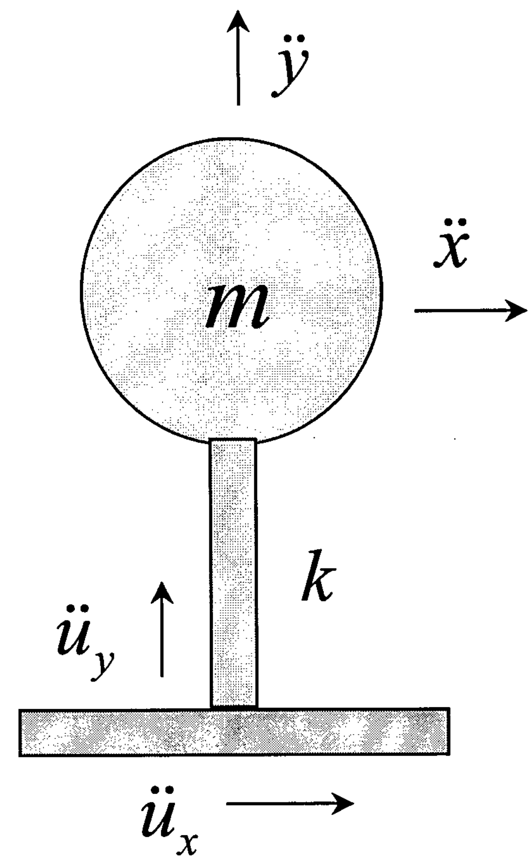
- *Two Scaling Possibilities:*

(1) $\omega_n = f(E)$

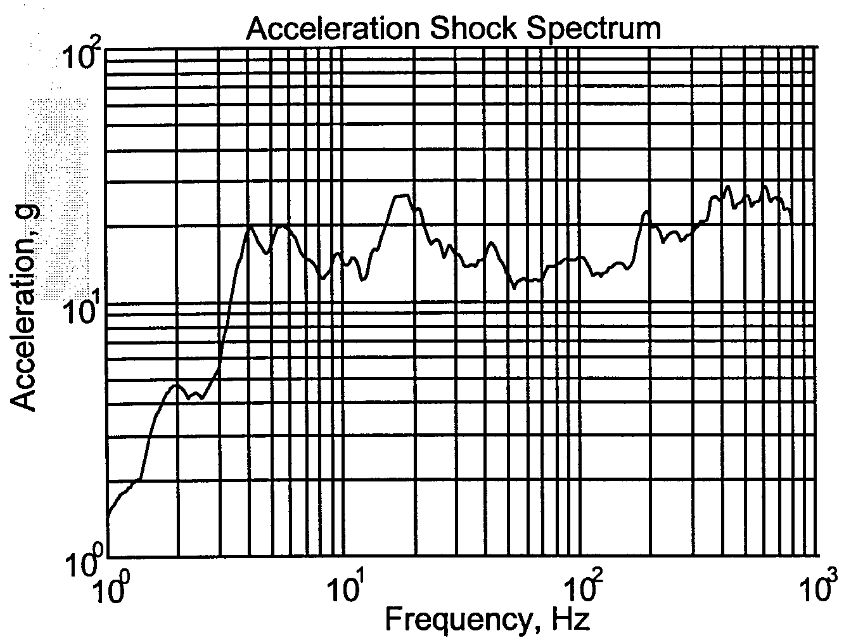
$$\sigma, \varepsilon \approx \delta \;\Rightarrow\; A / \omega_n^2$$

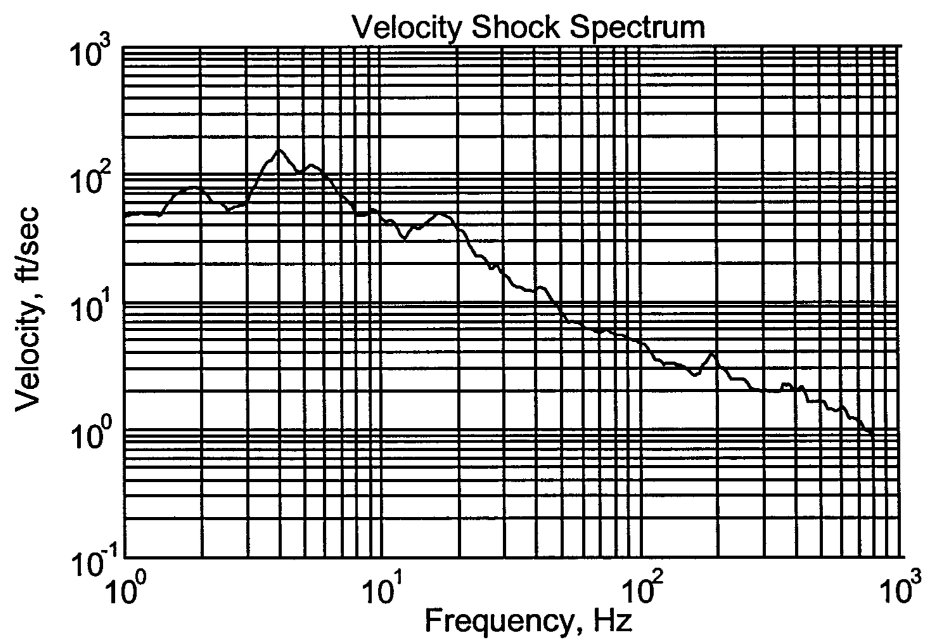
(2) $\omega_n = f(L)$

$$\left.\begin{array}{l} \omega_n \approx 1/L \\ \sigma, \varepsilon \approx L \end{array}\right\} \Rightarrow A / \omega_n$$



Acceleration versus Velocity Spectrum





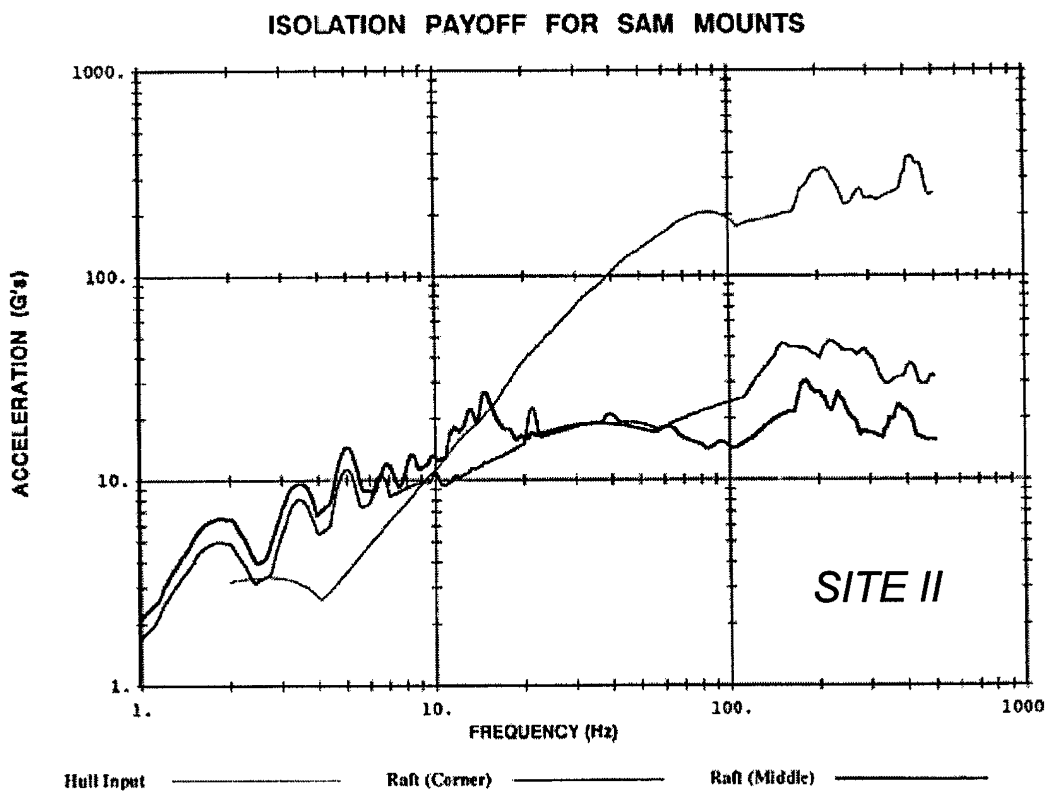
(1) What are the critical load-point cabinet dynamics for accurate prediction of truss response?

- "Isn't a lumped mass model good enough?"
- "Mounts can shift energy to higher frequencies."

(Names withheld by request.)

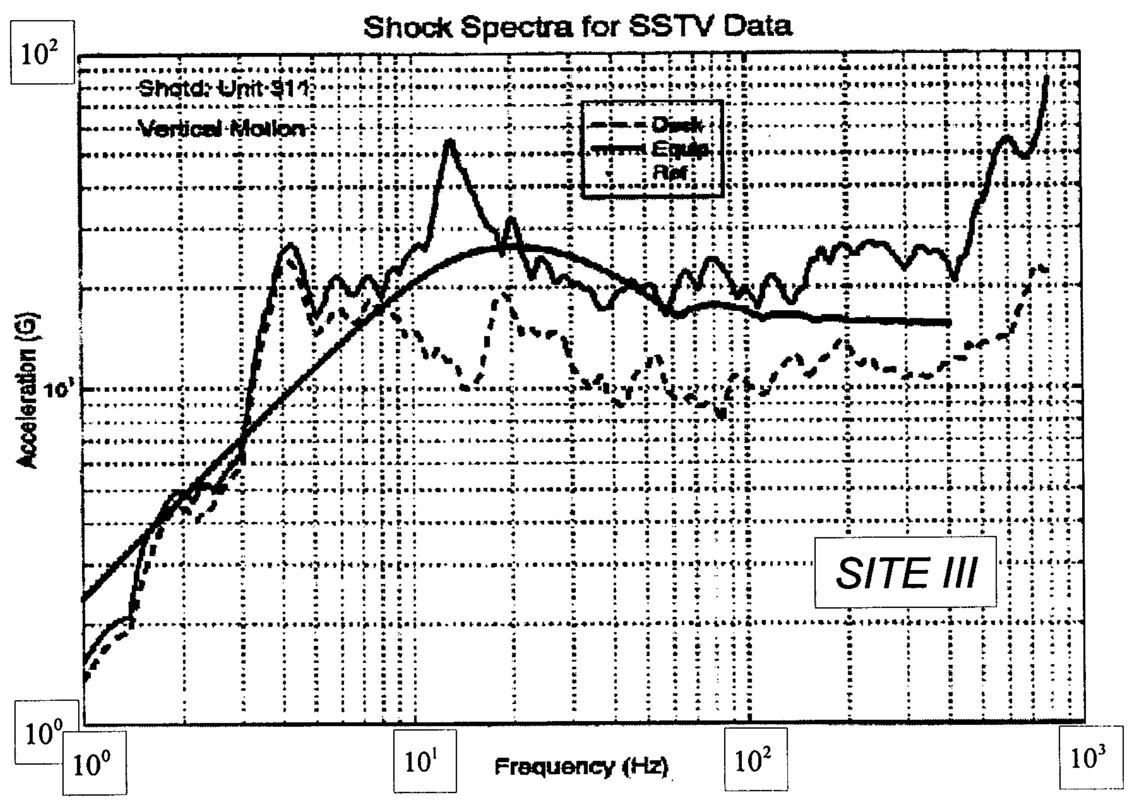
Effect of mount on shock spectrum

- *Mounts shift energy to lower frequencies.*
- *Crossover at 10-20 Hz.*



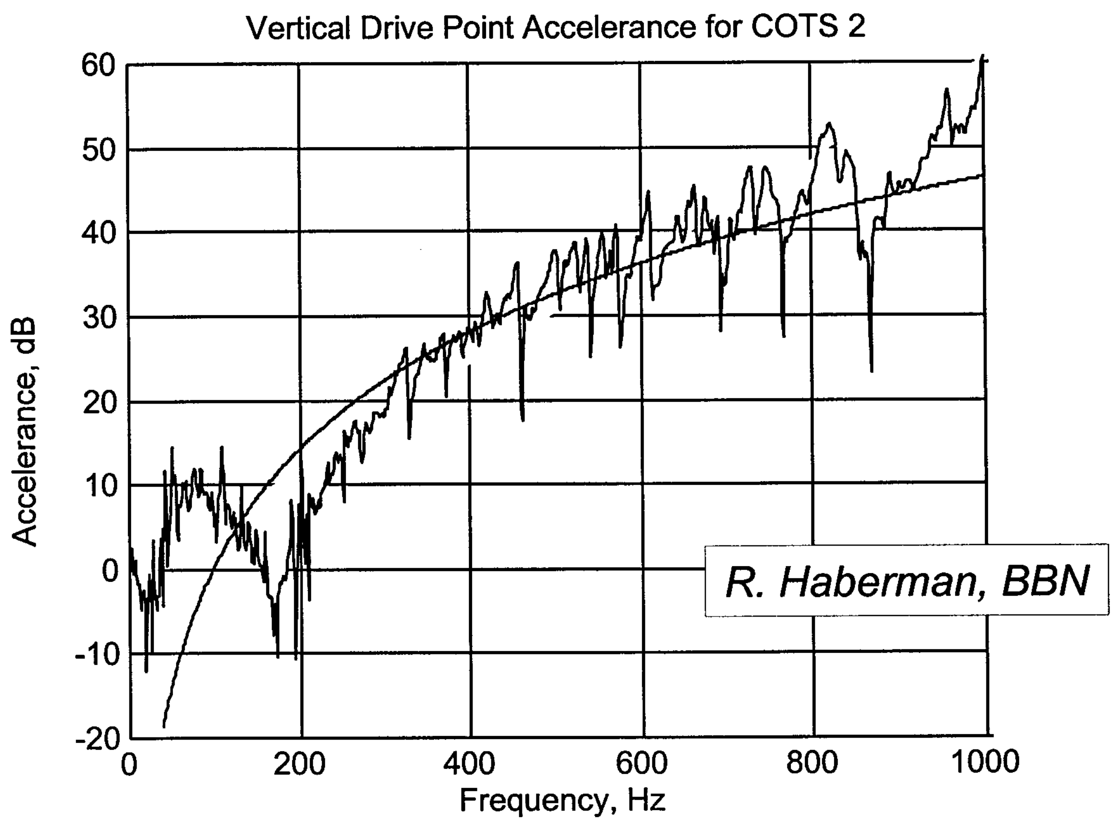
Lumped mass models?

- *Deck response attenuated by fixed-base cabinet mode at ~14Hz.*



Comparison with Cabinet Data

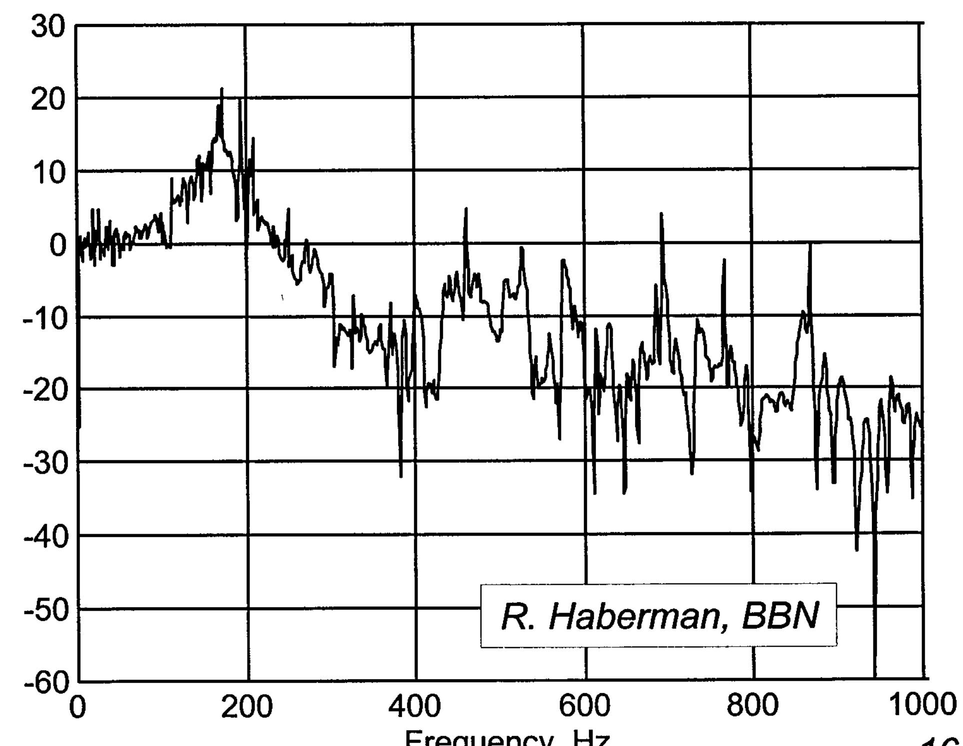
- COTS 2 Frequency Response:



(2) How does equipment response vary with location in a cabinet?

- *Transfer function for COTS 2 cabinet*
- *Use to produce shock spectrum within cabinet.*

$$G(\omega) = \frac{F[\ddot{y}(t)_{top}]}{F[\ddot{y}(t)_{bottom}]}$$



Research at Carderock - 1

- *FEM comparison of truss response with SITE II experimental data:*
  - *ETC truss model*
  - *Inputs are below-mount velocities.*
  - *Equipment models:*
    - *Lumped mass.*
    - *Teepee: center of mass preserved.*
    - *Shell elements: first bending mode preserved.*

Research at Carderock - 2

- *Build reduced-order truss model:*
  - *impedance model*
  - *FEM substructuring*
- *Input / output nodes :*
  - *mounts*
  - *array of cabinets*
- *Analytical models of cabinets and mounts.*

Reduced order truss model

- *Test bed for comparison of cabinet and mount models.*
- *Fuzzy approach to cabinet dynamics, bracing, and deck placement.*
- *Benefits:*
  - *Design of equipment emulators.*
  - *Design of shock acceptance tests.*

Summary

- *Navy goal: truss and mount design.*
- *Shock spectrum versus stress and strain.*
- *Myths of shock loading.*
- *Research at Carderock.*
- *University / Navy Lab interaction*

University / Navy Lab Interaction

- *What can universities do?*
  - *Experiment Design Partnerships*
  - *Data Analysis Partnerships*
- *What can Navy Labs do?*
  - *Improve documentation of results.*
  - *Preserve and transfer knowledge.*

Equipment Emulators

- *Benefits of using appropriate emulators:*
  - *Test realism*
  - *Cost savings*
- *Two questions to address:*

  (1) What are the critical load-point cabinet dynamics for accurate prediction of deck response?

  (2) How does equipment response vary with location in a cabinet?

(1) Cabinet's Contribution to Deck Response

- *Mass*
- *Local Stiffening*
- *Dynamics - acts as vibration absorber.*

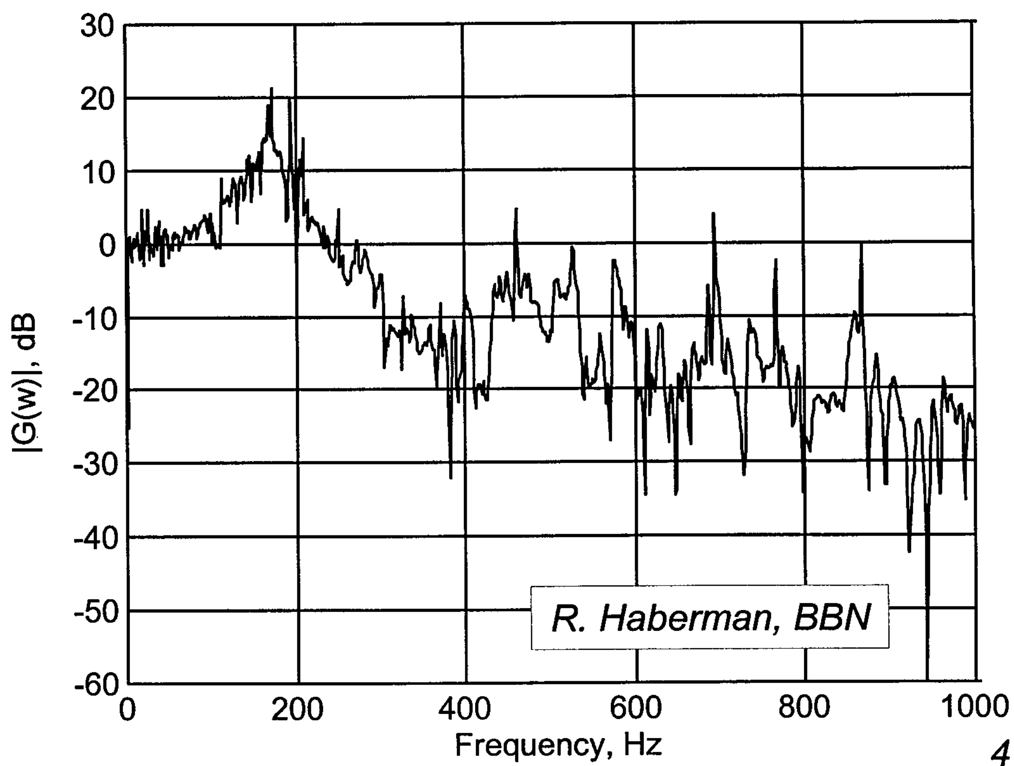
FEM comparison of truss response with SITE II experimental data

- *Time series and shock spectra*
- *Existing data:*
  - *EB:*
    - *5 cabinet models*
    - *deck response*
  - *ETC:*
    - *FEM shell element cabinet model*
    - *deck and equipment response*

(2) How does equipment response vary with location in a cabinet?

- *Transfer function for COTS 2 cabinet*
- *Can use to produce shock spectrum within cabinet.*

$$G(\omega) = \frac{F[\ddot{y}(t)_{top}]}{F[\ddot{y}(t)_{bottom}]}$$



Equipment modeling test bed

- *Impedance model of SITE II truss to be built by ETC.*
- *Input / output nodes :*
  - *mounts*
  - *array of cabinets*
- *To be used with analytical models of cabinets and mounts.*

Equipment modeling test bed - 2

- *Allows for systematic study of:*
  - *deck stiffening and dynamics due to equipment.*
  - *response at all locations within cabinet.*
  - *effect of mount on equipment response.*
- *Can employ fuzzy modeling approach to cabinet dynamics, bracing, and deck placement.*

Goals of Test bed

- *Develop design guidelines for shock trial equipment emulators.*
- *Develop fundamental understanding of overall cabinet response.*

Pierre Dupont
Boston University

03/09/98

Effectiveness of Box Beam Fill in Shock Mitigation

I wanted to follow up on my comments on how to gage the effectiveness of beam fill material on shock mitigation. In particular, I want to address three issues:

1. the effect of linear damping on shock mitigation in general,
2. assessing damping effectiveness of fill material through understanding fill damping mechanism, and
3. the effect of nonlinear damping on shock mitigation.

1. Effect of Linear Damping:

At the meeting, it was mentioned that the frequency of the mounts together with the truss is about 5 Hz. Note that the associated mode shape is essentially rigid-body motion of the truss on the mounts. Of course, from the acoustics point of view, the low mount frequency minimizes deck/truss response due to periodic higher-frequency forcing from the equipment. This in turn minimizes hull excitation and consequently acoustic radiation.

The goal is similar for shock loading. By making the "mount frequency" much smaller than the truss structural frequencies, you create an impedance mismatch between the mount and the truss so as to minimize the transfer of energy into modes associated with truss deformation. Put another way, you reduce the modal participation factors of the truss modes. According to Bill Gilbert, the design goal was a 10:1 separation between truss frequencies and the mount frequency. In practice, a separation of about 5:1 is achieved. Assuming a 5 Hz mount frequency, this indicates that the lower end of truss frequencies is about 25 Hz. Again according to Bill Gilbert, general DDAM design practice for shock loading considers frequencies up to 250 Hz to be important.

ONR-speak: Does box-beam fill material help to mitigate shock response?

Suggested Translation: Does the fill material increase damping in the modal frequency range (~25-250 Hz) of the truss?

Caveats:

1. From the viewpoint of equipment protection, we want to avoid having the deck respond in the frequency range of the equipment. BBN's vibration tests of COTS equipment indicate a significant frequency response around 15-20 Hz. Thus, it may be appropriate to consider the effect of bead damping in the range 15-250 Hz.
2. Damping due to fill material is only significant during beam bending. My guess is that truss mode shapes primarily involve the flexure of beam members, but this might be given further thought.

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03/09/98

Observations:

1. For arbitrary shock inputs, additional damping always lowers the shock spectrum (g-levels) across all frequencies.
2. To quantify the short- versus long-time effect of damping on shock response, an example from the *Shock and Vibration Handbook* (page 8.52, 4th edition) is instructive. Under steady sinusoidal forcing, an increase from 1% to 10% of critical damping in a single degree of freedom system results in a tenfold decrease in response amplitude at resonance. When the same system is acted upon by a half-cycle sine pulse with a (worst-case) duration of half the system period, the same increase in damping only reduces the maximum response by 9%.

2. Assessing Damping Effectiveness through Understanding of the Fill Damping Mechanism:

Modeling efforts suggest that bead vibration in the plane of the beam cross section is primarily responsible for box beam damping. It is my understanding that, in addition to the report by David Feit and David Warwick, Nate Martin of BBN presented modeling results at several CRADLE program reviews a year or two ago. Copies of the overheads should be in the Carderock files -- perhaps of Dave Warwick. (I don't have copies myslef.) As described to me, BBN's results show that fill damping in a cross section reaches a maximum at bead resonance and then falls to a high-frequency asymptote which depends on the impedance mismatch between the steel of the beam and the fill material.

It may be useful (and quick) to compare the frequency range of significant fill damping obtained by BBN with the desired range described above.

3. Effect of Nonlinear Damping on Shock Mitigation:

Items 1 and 2 above relate to data which could be obtained from vibration tests. Perhaps it is obvious that the real benefit to be gained from shock table experiments would be to quantify the *nonlinear* damping effects of fill material.

Professor Al Ferri at Georgia Tech has been funded by ONR to determine optimal nonlinear mount behavior for shock mitigation. For many types of shock inputs, the optimal mount force has a velocity dependence similar to that of Coulomb friction. Consequently, I don't think that this work is directly (and quickly) applicable to deciding on the efficacy of fill damping for shock.

Given rattle space limitations, it *is* desirable for damping to increase nonlinearly for large deformations of the truss (or mounts). This provides for a smooth snubbing effect. In this way, damage caused by high-velocity impact with hard snubbers can be avoided.

Design of COTS Emulators for Shock

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March 9, 1998

1.0 Introduction

To achieve lower costs and higher performance, a thrust is underway to replace MIL-SPEC computer equipment and cabinets with COTS equipment and cabinets. At the same time, shock and acoustic performance levels must be maintained. Consequently, the design focus has shifted away from the equipment and cabinets and entirely onto the deck and mounts.

When evaluating deck response under vibration or shock loading, it has long been recognized that feedback from the equipment dynamics has a significant effect on the response. In particular, this means that equipment fixed-base frequencies are not so high that rigid mass equipment representations are adequate for shock trials. Furthermore, COTS equipment is, in general, more flexible than the MIL-SPEC equivalent, indicating that its fixed-base frequencies will be even lower.

In order to adequately evaluate a truss or mount design through scale model trials, it is therefore important to build equipment emulators which embody the salient dynamic properties. This document provides an overview of emulator design guidelines for shock. This material is preceded by a brief introduction to shock spectra and shock qualification standards. A detailed tutorial on shock spectra appears in the appendix.

2.0 Shock Spectra

A shock event can be described in a number of ways. For example, the time or frequency domain representations of motion histories can be used. These quantities are typically among those directly measured. Alternatively, a shock event can be described in terms of its effect on structures, e.g., the maximum stress experienced by some part of the structure. When comparing the severity of shock events, the latter approach is often preferable since it expresses severity in terms directly relevant to structural design and survivability.

Shock spectra belong to the second class of representations. For a given base motion history, a shock spectrum is a plot of the *maximum* relative displacement, δ_{max}, versus natural frequency, ω, of an oscillator attached to the base.[1] Assuming linear, elastic behavior, the maximum relative displacement is proportional to the maximum stress and strain ($\sigma = E\varepsilon$, $\varepsilon = \Delta l / l$) experienced by attached equipment of that natural frequency. On a shock spectrum plot, the vertical units are often scaled by ω or ω^2 / g to obtain units of velocity and acceleration (expressed in g's), respectively. Given that the forcing is an arbitrary base motion history, these quantities are not, in general, the actual velocity and acceleration of the oscillator. In the literature, they are appropriately referred to as the pseudo velocity and pseudo acceleration.

Note that since the shock spectrum is computed using the *maximum* relative displacement, it is *not,* in general, a scaled version of the Fourier transform of the base motion. For the same reason, the inverse map from shock spectrum to base motion is not unique. This is discussed in more detail in the appendix.

The reason that shock spectra are often plotted in acceleration units stems from their traditional use in structural design. Stress in a particular fixed-base mode of a complicated structure is computed from the equivalent static load obtained by multiplying the modal mass by the modal acceleration. The latter quantity is read off the design shock spectrum at the modal frequency. Total structural stress is estimated using a modal summation, e.g., the NRL Sum used in DDAM.

3.0 Shock Qualification:

NAVSEA qualifies COTS equipment and decks using shock spectra. Fully-loaded equipment cabinets are mounted on shock tables and a specific base time history is applied, e.g., a 15g half-sine acceleration pulse of 40 msec duration. If the equipment remains functional, it is judged to be able to withstand any base motion whose shock spectrum is less than or equal to the shock spectrum associated with the test input. Thus, to qualify the deck on which this equipment is to be mounted, one must show that deck motion in response to expected external explosions produces a shock spectra less than or equal to that of the test input.

While it is the deck motion which induces strain in the equipment, the equipment dynamics can greatly influence the deck motion. In particular, the equipment acts as a vibration absorber at each of its fixed-base frequencies. Consequently, the deck motion and associated shock spectrum are significantly reduced from the motion which would occur with an equal amount of rigid mass. Analytical examples have shown that modal masses representing 2-5% of the total system mass can dramatically reduce shock spectrum peaks (which lie at modal frequencies of the entire system, i.e., the truss and equipment).

[1] In the remainder of this document, ω refers to the natural frequency of a simple oscillator or to a fixed-base frequency of deck-mounted equipment.

DDAM (Dynamic Design and Analysis Method) was developed for the design of foundations for heavy, aft-end equipment. As described above, total stress is estimated using a sum of equivalent static loads applied to the fixed-base modes. The relevant static accelerations obtained from a design shock spectrum are thus the values at the fixed-base frequencies. Given the vibration absorber analogy presented above, the shock spectrum values pertinent to structural design typically *lie at the minima* of an experimentally derived shock spectrum.

The goal of emulator design is to produce scaled deck motion which is accurate with respect to those properties critical to the design and qualification process. Thus, an emulator designed for DDAM purposes must predict the shock spectra minima by accurate representation of the primary fixed-base modal frequencies and masses. In contrast, COTS equipment qualification requires accurate prediction of the shock spectrum over the entire frequency range of interest. The justification for this approach stems from the fact that it is unknown which COTS components will fail first. These components could be of insignificant mass in comparison to total equipment mass and consequently lie in proximity to a shock spectrum maximum.

As a result, COTS emulator design and deck qualification are governed by accurate prediction of shock spectrum maxima. Analytically, this suggests that the COTS emulator design problem may be more difficult than the DDAM design problem. There is some evidence, for example, that damping has a small effect on shock spectrum minima, but a large effect on shock spectra maxima. Similarly, the sensitivity of spectrum maxima to relatively small amounts of modal mass indicates that modal truncation rules such as those employed in DDAM may deserve further attention.

4.0 Design Guidelines:

The following paragraphs provide a set of design guidelines for COTS emulators. The list of dynamic properties to be reproduced in the emulators is straightforward. What is less obvious is the level of accuracy necessary for effective emulation. These guidelines should consequently be viewed as preliminary. It is recommended that full characterization of the shock emulators ultimately employed in the upcoming quarter scale trials be undertaken. Analysis of this data, together with that of the quarter scale trials, provides an opportunity to validate these guidelines and assumptions.

Prior work in the design of scaled equipment emulators for shock appears to be minimal. Some guidance is provided by DDAM as well as by MIL-STD 167, for vibration testing, and MIL-S 901, for shock testing. In particular, the following assumptions are made in equipment qualification and design:

1. Equipment design, testing and qualification is done by considering input motions and the resulting shock spectra independently along three rectilinear orientation axes: vertical, athwartship and fore and aft.

2. While the method of attachment is considered, the potential of deck flexibility between attachment points is ignored.

4.1 Dynamic Properties

The COTS properties to be reproduced in equipment emulators fall into four categories: modal properties, damping, method of attachment and nonlinearities. Each is discussed separately below. Design recommendations are then summarized in section 4.2.

4.1.1 Modal Properties

In the most general sense, the impedance matrix for the emulator attachment points should match that of a typical full-scale COTS cabinet – at least over the frequency range of interest for shock. The low end of the frequency range is dictated by the COTS equipment. BBN's vibration tests indicate a significant modal response around 15-20 Hz. General DDAM design practice considers frequencies up to 250 Hz to be important for shock.[2]

In the discussion that follows, it is assumed that modal analysis techniques can be used to convert measurements of impedance to an equivalent system of attached oscillators. The mass of each oscillator will be referred to as the "modal mass" and the resonant frequency of each as the "modal frequency".[3] These frequencies correspond to the fixed-base frequencies of the equipment.

In practice, an emulator should reproduce all significant modal frequencies and masses for motion in the three principle directions over the frequency range of interest. DDAM, for example, specifies that those modes included in an equipment model should contain at least 80% of the total modal mass lying in that frequency band. Furthermore, a finite element model must include all modes representing 1% or more of this total modal mass. Simple analytical examples have shown that modal masses of 2-5% of the total mass (truss and equipment) significantly affect the shock spectrum. Note that if an array of similar cabinets is present on a deck, it is necessary to add the modal mass contribution from each in order to assess the importance of including a particular mode.

The significant modes will not, in general, account for all the mass of the emulator and so the question arises as to how the remaining mass should be distributed with frequency. Clearly, mass tied to modal frequencies above the range of interest can be treated as rigid. The situation is not so clear at lower frequencies. In contrast to MIL-SPEC cabinets, COTS cabinets exhibit high modal density. This suggests that a fuzzy structures approach

---

[2] Truss frequencies fall within this range. According to Bill Gilbert, the design goal is a 10:1 separation between truss frequencies and the mount frequency. In practice, a separation of about 5:1 is achieved. Assuming a 5 Hz mount frequency, this indicates that the lower end of truss frequencies is about 25 Hz.
[3] This is the notion of modal mass introduced in G. O'Hara and P. Cunniff 1963, *Elements of Normal Mode Theory*, NRL Report 6002.

to mass distribution may be appropriate.[4] In COTS, it is the plates composing the sides of the cabinets which produce the high modal density. The same mechanism can be used to achieve the correct modal mass distribution in emulators.

4.1.2 Damping

The role of damping in shock modeling has traditionally been considered to be minor. An example from the *Shock and Vibration Handbook* is instructive in understanding this viewpoint.[5] Under steady sinusoidal forcing, an increase from 1% to 10% of critical damping in a single degree of freedom system results in a tenfold decrease in response amplitude at resonance. When the same system is acted upon by a half-cycle sine pulse with a (worst-case) duration of half the system period, the same increase in damping only reduces the maximum response by 9%.[6] In actual UNDEX events, however, repeated bubble pulse loading can occur. This situation is closer to that of steady forcing and, thus, damping plays an important role in limiting the maximum response.

4.1.2 Method of Attachment

The mechanical connection between equipment and the deck can be a significant source of flexibility and, in some cases, damping. It is therefore important to accurately reproduce these properties in the scaled system. If any equipment is braced, the properties of the bracing should be incorporated in the scaled system as well.

4.1.3 Nonlinearities

Shipboard equipment is classified according to how its failure affects ship survivability and military capability. Shock standards for less critical equipment allow for some plastic deformation and/or failure. When equipment does yield or fail, its dynamic interaction with the deck changes. This, in turn, affects the loading of other equipment. In order for a scale shock trial to be realistic, equipment emulators should yield and fail at loading levels corresponding to those of their full scale counterparts.

COTS cabinets are proposed to house electronic equipment which, in general, must survive shock loading undamaged. Furthermore, it is expected that the quarter scale trials will require repeated shock loading of the truss and equipment. Consequently, the emulators should be designed to perform elastically under repeated loading of the anticipated level.

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[4] While a fuzzy mass distribution will increase apparent damping of the equipment, the direct effect on the shock spectrum is unknown. This is a topic of current study.

[5] C. Harris, Editor 1996, McGraw Hill, New York, 4th edition, page 8.52.

[6] This appears to contradict observations by Cunniff and O'Hara that damping can significantly affect shock spectra maxima. This topic is currently under investigation.

4.2 Design Recommendations

Preliminary design recommendations for COTS emulators are presented below.

Modal Properties - For each COTS equipment modal frequency in the range 15-250 Hz.:

1. Order the modes by mass and include a number sufficient to contain at least 80% of the total modal mass in this frequency range.
2. Add the modal masses for all of the cabinets on a deck and incorporate any additional modes containing 5% or more of the total mass (truss and equipment).
3. If the equipment exhibits high modal density in this frequency range, account for it in the emulator design.
4. The modal frequency of the remaining mass should lie above the range of interest and can be included in the emulator as rigid mass, i.e., possessing any modal frequency sufficiently above 250 Hz.

Damping - Modal damping should be estimated from mobility measurements and incorporated in the emulator design.

Method of Attachment - The proposed methods of attachment for actual COTS equipment should be identified and their mechanical properties measured and scaled for inclusion in the emulator design.

Nonlinearities - The emulators should be designed to survive elastically at least several hundred cycles at the anticipated loading level. To this end:

1. Perform FEM analysis to check for potential low-cycle fatigue damage.
2. Perform repeated shock table loading of a single cabinet emulator and inspect for plastic deformation, cracking, etc. Modify emulator design as needed.

6.0 Acknowledgment

Information presented here was obtained from a variety of people and documents. Special thanks is extended to Bill Gilbert and Paul Young of NSWCCD.

Appendix A - Shock Spectra

Shock spectra were first proposed by Biot in the 1940's as a means to understand the severity of an earthquake in terms of the loading experienced by a building. In the seismic community, it is assumed that the building does not affect the ground motion. Consequently, seismic design shock spectra are derived as envelopes of the spectra obtained for various earthquakes. In contrast, it was observed during the 1950's that for

Naval applications, feedback from deck-mounted structures had a significant effect on deck response and thus on the shock spectrum.

The shock spectrum, $S(\omega,\zeta)$, is defined as the maximum relative displacement, $\delta(t)$, of a fictitious oscillator of natural frequency ω subject to the base motion $\ddot{u}(t)$ as shown in Figure 1. The equation describing displacement of the oscillator is given by the Duhamel integral of the following equation.

$$\delta(t) = \frac{1}{\omega\sqrt{1-\zeta^2}} \int_0^t \ddot{u}(\tau) e^{-\zeta\omega(t-\tau)} \sin\omega\sqrt{1-\zeta^2}(t-\tau)\,d\tau$$

Here, $\zeta = c/2m\omega$ is the damping ratio of the oscillator.

The shock spectrum is plotted in terms of displacement, pseudo velocity or pseudo acceleration as given by the following three expressions, respectively.

$$S(\omega,\zeta) = \max_{0 \le t < \infty} \delta(t)$$

$$V(\omega,\zeta) = \omega S(\omega,\zeta)$$

$$A(\omega,\zeta) = \frac{\omega^2}{g} S(\omega,\zeta)$$

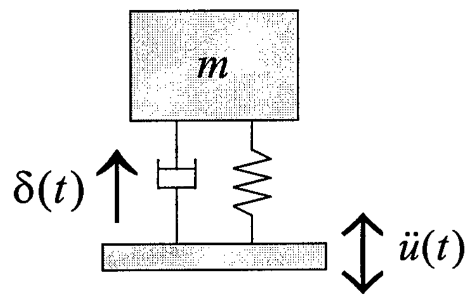

Figure 1 - Oscillator for computing shock spectrum.

The fictitious oscillator used to compute the shock spectrum behaves as if it is massless in that its presence does not affect the base motion. It can be shown, however, that its displacement coincides with the actual displacement of real deck-mounted equipment of the same natural frequency. At frequencies which do not coincide with fixed-base equipment frequencies, the response of the fictitious oscillator overestimates the response of equipment with appreciable mass.

Clearly, an actual shock loading involves motion in more than one direction. While one could envision defining shock spectra with several degrees of freedom, the tradition in the seismic and Naval communities has been to consider shock spectra independently for the three translational directions of motion. Furthermore, in DDAM, the equipment base is considered rigid. Thus, only a single attachment point need be considered. While no further mention will be made of these assumptions here, this is not meant to imply that they are necessarily appropriate for COTS equipment mounted on isolated deck modules.

A.1 Relationship Between the Shock Spectrum and the Fourier Spectrum of the Base Motion

It is natural to question what value is added by the shock spectrum in comparison to simply computing the Fourier spectrum of the base motion. Clearly, the Duhamel convolution integral is equivalent in the Fourier domain to

$$\Delta(\omega) = G(\omega)\ddot{U}(\omega)$$

where $\ddot{U}(\omega)$ is the Fourier spectrum of the base acceleration and $G(\omega)$ is the transform of the unit acceleration impulse response of the oscillator. Is then the shock spectrum not simply a frequency-weighted function of $\ddot{U}(\omega)$? The key point to understanding the difference here is to note that the shock spectrum involves taking a maximum *in the time domain.* So while there is a one-to-one relationship between $\Delta(\omega)$ and $\ddot{U}(\omega)$ in the equation above, only the forward mapping from $\ddot{U}(\omega)$ to $S(\omega)$ is uniquely defined.

This is not to suggest, though, that partial information about $S(\omega)$ cannot be gleaned directly from $\ddot{U}(\omega)$. To understand this connection, let us define the *residual shock spectrum* as the maximum relative displacement which occurs after the shock input has ended.[7] This is given by

$$S_r = \max_{t_s < t < \infty} \delta(t)$$

where the time duration of the shock input extends from $t = 0$ to $t = t_s$. By comparing the definitions, it is clear that the residual shock spectrum provides a lower bound on the shock spectrum.

Let us consider the undamped case, i.e., when $\zeta = 0$. We can rewrite the convolution integral in this case to be

$$\delta(\omega, t) = \frac{1}{\omega}\mathrm{Im}\left\{\int_0^t \ddot{u}(\tau)e^{j\omega(t-\tau)}d\tau\right\}$$

$$= \frac{1}{\omega}\mathrm{Im}\left\{e^{j\omega t}\int_0^t \ddot{u}(\tau)e^{-j\omega\tau}d\tau\right\}$$

We can define

[7] For this discussion, the shock input corresponds to the base velocity or acceleration which is assumed to go to zero fairly quickly. In some contexts, the shock input may refer to pressure pulses on the hull. In these cases, significant deck motion may continue after pressure transients have effectively subsided. Thus, the time period over which the residual spectrum is computed will include a portion of nonzero base motion.

$$(\omega, t) = \int_0^t \ddot{u}(\tau) e^{-j\omega\tau}\, d\tau$$

as the *running Fourier spectrum* and note that for $t > t_s$, $(\omega, t) = \ddot{U}(\omega)$, the Fourier spectrum of the input acceleration. Then we can write,

$$\delta(\omega, t) = \frac{1}{\omega} \mathrm{Im}\left\{ e^{j\omega t} \mathrm{F}\ (\omega, t) \right\}$$

Defining the time-dependent magnitude and phase of the running Fourier spectrum in the usual way, we can write $(\omega, t) = F(\omega, t) e^{j\theta(\omega, t)}$. The relative displacement can now be written as

$$\delta(\omega, t) = \frac{1}{\omega} F(\omega, t) \sin\{\omega t + \theta(\omega, t)\}$$

Recalling the definition of residual shock spectrum and noting that F and θ are independent of time for $t > t_s$, we now have our result that the undamped residual shock spectrum is related to the magnitude of the base motion spectrum by

$$S_r = \frac{1}{\omega} mag\{\ddot{U}(\omega)\}$$

Similarly, it can be shown that in terms of base input velocity,

$$S_r = mag\{\dot{U}(\omega)\}$$

Recalling that $S(\omega) \geq S_r(\omega)$, we conclude that the Fourier spectrum of base velocity provides a lower bound on the undamped shock spectrum.

It is of interest to know if the residual and regular shock spectrum are equal over certain frequency ranges. This is equivalent to asking when and if the maximum oscillator displacement occurs after the shock input has terminated, i.e., for $t > t_s$. O'Hara suggests that the spectra tend to agree at the combined system frequencies.[8] He argues that, due to "resonant buildup" at these frequencies, the maximum displacement will normally occur at the end of the shock input. While this is clearly not true for arbitrary shock inputs, it may well be shown correct for a large class of inputs when filtered by the mechanical system's transfer function.

Assuming that the shock and residual spectra agree at the system frequencies, this means that they agree at their maxima. As indicated earlier, it is the spectrum maxima which are of interest in qualifying COTS equipment. In this case, deck qualification would be equivalent to ensuring that the peaks of the base velocity Fourier spectrum lie below the deck qualification shock spectrum.

[8] George O'Hara 1959, "Impedance and Shock Spectra," *JASA* 31:10, 1300-1303.

SITE raft velocity data was used to compute the shock and residual velocity spectra shown below. Figure 3 depicts the undamped case just described. It can be seen that the residual spectrum does tend to reproduce the shock spectrum maxima corresponding to the system natural frequencies. The spectra for 5% damping are plotted in Figure 4. Clearly, the peaks of the undamped residual spectrum of Figure 3 (corresponding to the Fourier transform of the input velocity) overpredict the damped shock spectrum of Figure 4 through most of the frequency range. This indicates that the Fourier transform of the input motion is not a good indicator of equipment stress levels when the equipment possesses a modest amount of damping.[9]

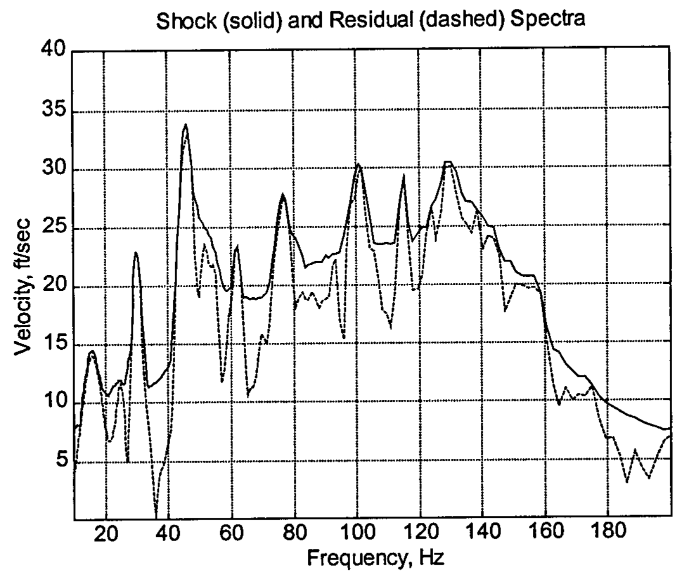

Figure 3 - Comparison of undamped shock and residual spectra.

[9] Note that since the base motion is prescribed, this example does not assess the effect of equipment (or emulator) damping on the base motion. This is a topic of current study.

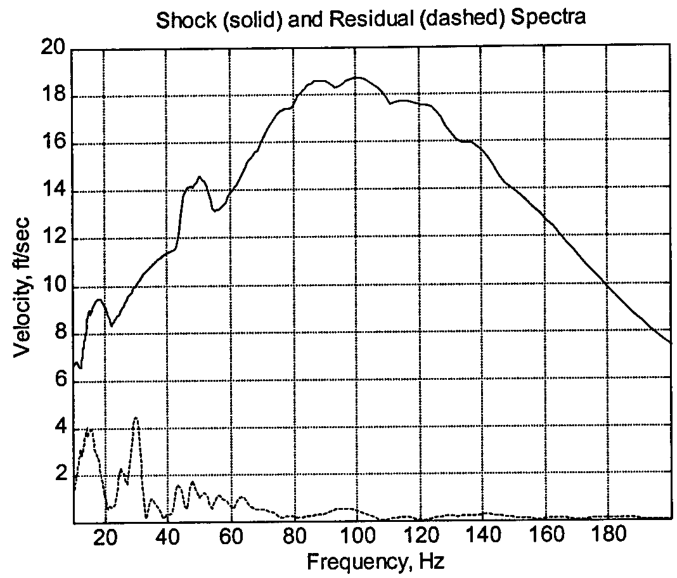

Figure 4 - Shock and residual spectra for 5% damping.

EQUIPMENT EMULATION FOR SCALE SHOCK TRIALS

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Research Objectives

In a variety of situations, an undesired shock excitation is applied to a master structure which supports shock-sensitive equipment. Often, one wishes to design and test a master structure which attenuates the amount of shock energy transmitted to the attached equipment. In scaled testing of new designs, a major task is to design and construct "equipment emulators" – inexpensive mechanical systems which approximately mimic the dynamic behavior of the actual full-scale equipment as seen by the base structure.

The objectives of this research are to develop new methodologies for designing equipment emulators, assessing their fidelity, and interpreting test data taken in the presence of imperfect emulators. These concepts are applied to the emulation of electronic equipment for the testing of novel ship deck structures.

Current Research Approach

In the design of equipment emulators, current naval practice is to build a nominal model that attempts to match several fixed-base equipment modes. Subsequently, an ad-hoc, iterative approach is used to tune the design by comparing attachment point mobility of the emulator with that of the actual equipment. While this method is a general one, it is labor intensive and does not provide any means of interpreting how emulation errors affect the shock environment of the deck structure. Furthermore, as complex subsystems, such as COTS equipment cabinets, can be of high modal density, it is unclear whether or not low modal approximations yield appropriate models.

To address these issues, we have proposed criteria which express emulator error and sensitivity to emulator error in the context of deck velocity and its associated shock spectra. This approach provides several advantages:

1. Equipment modeling effort can be concentrated on those frequencies that are most important to the testing of a particular deck structure.
2. An emulator can be designed to a specified level of error in deck velocity.
3. Given models of the actual and emulated equipment, shock trial data can be corrected for emulator error.
4. Sensitivity can be used to guide the refinement of a nominal emulator design.

These concepts are undergoing further development for application to the Navy's ¼ scale shock trials planned for FY 1998.

Significant Research Findings

While it is accepted that equipment emulators are needed for the acoustic evaluation of naval structures, there was some sentiment within the Navy community that equipment modeling using dead mass may be sufficient for shock. This issue was investigated using data from the full-scale shock trials of SITE III. While DDAM, used to design mounts and foundations for heavy equipment, indicates that equipment acts like a vibration absorber at its fixed-base frequencies, this effect had not been validated for forward compartment equipment such as electronic cabinets. Figure 1 compares the shock spectrum computed from velocity measured at the deck and at the top of a COTS cabinet. It is clearly seen that, for the shock event, deck response is reduced at 14 Hz, which corresponds to a fixed-base frequency of the cabinet. Data such as this provides conclusive evidence that dynamic emulation for shock qualification is necessary.

Current practice assumes commercial electronic equipment can withstand a shock loading corresponding to a half-sine acceleration pulse with amplitude 15 g's and duration 40 msec. Consequently, shock qualification of a deck requires that its shock spectrum not exceed that of the 15g-40msec pulse. What this fails to consider, however, is that shock response varies within a cabinet. This topic was studied using full-scale data from COTS cabinet vibration tests. Figure 2 depicts the experimentally derived transfer function relating acceleration at the top of a cabinet to that at the bottom. Significant amplification occurs in the range of 100-200 Hz indicating that the deck response of an unbraced cabinet can significantly underestimate actual equipment shock loading. Viewed together with the full-scale shock trial data of SITE III, it is clear that cabinet bracing and even the use of structurally integrated enclosures (SIE's) can reduce the overall shock loading within a cabinet or enclosure.

Navy plans call for ¼ scale shock testing of a three-deck truss structure supported by mounts during FY 1998. These plans call for the use of existing emulators developed for acoustic testing of a truss. Recent research efforts have been directed to providing NSWCCD personnel with guidance in the planning and interpretation of these experiments. Given that the emulators were designed for another program in an ad-hoc manner, there are two practical issues to be addressed:

1. Emulator characterization: How will deck response compare with that of perfectly scaled equipment, i.e., what is the emulation error?
2. Emulator modification: Is a particular emulator design change worth the cost, i.e., how sensitive is deck velocity to emulator modification?

These questions relate to the concepts of error and sensitivity, respectively. We have derived expressions for both quantities and have shown that, in terms of truss and mount properties, they depend solely on deck impedance at the attachment points. For example, emulation error, expressed in terms of deck velocity at the equipment base is given by

$$E = v_{2a} - v_{2e} = \left(\left[Z_a + Z_{22}\right]^{-1}\left[Z_e + Z_{22}\right] - I\right)v_{2e}$$

Error, E, is expressed as the difference between the deck velocity which would have been obtained using the actual equipment, v_{2a}, and the velocity obtained using equipment emulators, v_{2e}. Z_a, Z_e, and Z_{22} represent the attachment-point impedance matrices of the actual equipment, scaled equipment, and truss, respectively. Note that this expression does not depend on the mount to deck transmission impedance.

Figure 3 depicts an experimental plan for addressing these issues in the context of the ¼ scale shock trials. Here, v_1 represents the vector of hull velocities imposed on the truss while v_2 continues to represent deck velocity at the equipment attachment points. The top row of ellipses in the figure represents pre-trial experiments we have proposed to NSWCCD. Using the emulator, equipment and truss properties so obtained, it will be possible to correct shock trial deck velocity for emulation error. It will also be possible to suggest modifications of emulator parameters that will produce the greatest reduction in emulation error.

Relevance to the Navy

For the first ten months of this one-year grant, the PI was in residence at the Naval Surface Warfare Center, Carderock Division. This made it possible for the PI to familiarize himself with the unique set of resources available at Carderock, to interact extensively with Navy researchers, and to identify critical research issues.

This period was marked by significant change at Carderock. All shock-related activities were consolidated within the Advanced Structures Program under Mr. William Martin. During this period, the PI worked closely with Dr. Liming Salvino on the shock aspects of this program. The research objectives presented here as well as the experimental plan of Figure 3 were formulated to directly address the research needs of the program. The PI continues to attend program review meetings at Carderock and to provide timely advice regarding experimental design and analysis.

Technical Memoranda

1. P. Dupont, "Recommendations Relating to Advanced Structures Review Meeting of 8 October, 1997," submitted to W. Martin, NSWCCD.
2. P. Dupont, "Design of COTS Emulators for Shock," submitted to W. Martin, NSWCCD and G. Main, ONR, August 1, 1997.
3. P. Dupont, "Effectiveness of Box Beam Fill in Shock Mitigation," submitted to L. Salvino and E. O'Neil, NSWCCD, May 30, 1997.

Presentations

1. P. Dupont and G. McDaniel, "Towards a design methodology for equipment emulators in the shock testing of large structures," to be presented at the 135th Meeting of the Acoustical Society of America, Seattle, WA, June 1998.
2. P. Dupont, "Equipment modeling for scale shock trials," seminar, Naval Surface Warfare Center, Carderock Division, March 1997.
3. P. Dupont, "Equipment emulators for UNDEX testing," Office of Naval Research Program Review of Basic and Applied Research in Structural Dynamics and Structural Acoustics, Austin, Texas, February 18-21, 1997.

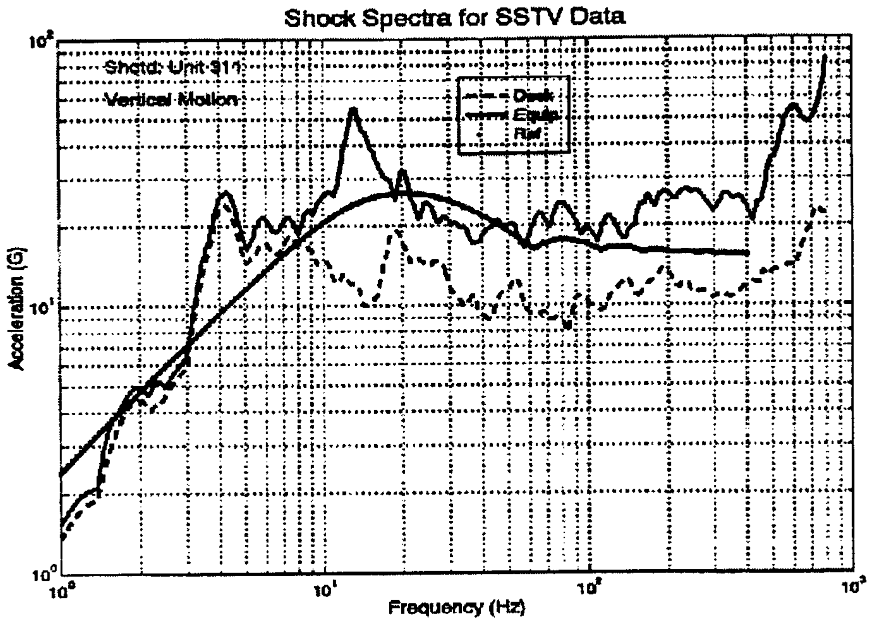

Figure 1 – Shock spectra from full-scale SITE III testing. The dashed curve is the deck-level response while the higher solid curve represents the response at the top of a commercial off-the-shelf (COTS) electronics cabinet. The peak in the solid curve at ~14Hz indicates that the equipment has absorbed a significant amount of energy at this frequency. The dip in the dashed curve at this frequency reflects the resulting attenuation of deck response.

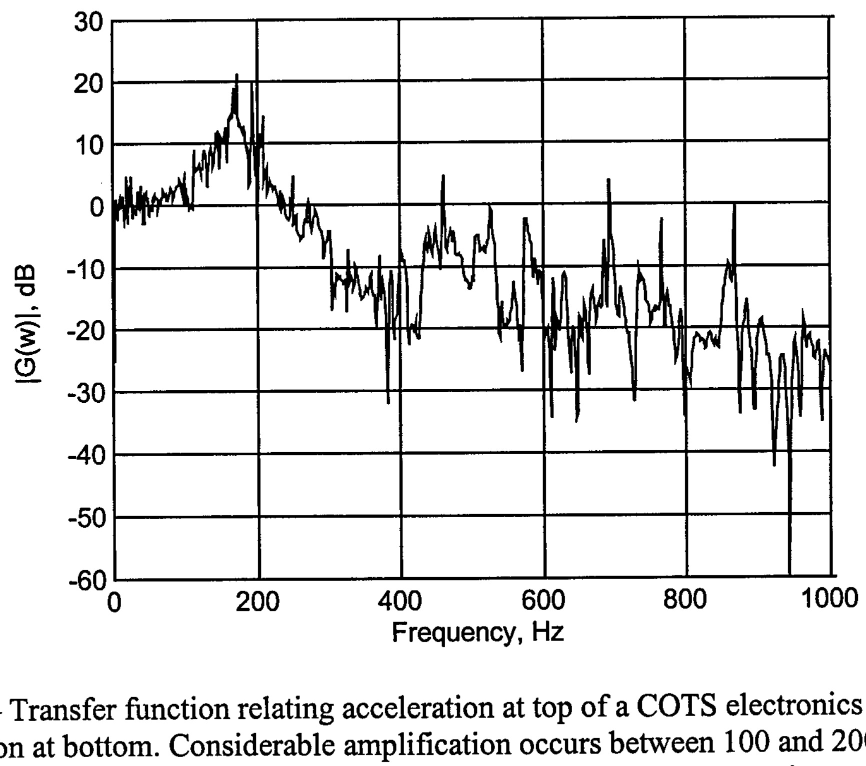

Figure 2 – Transfer function relating acceleration at top of a COTS electronics cabinet to acceleration at bottom. Considerable amplification occurs between 100 and 200 Hz indicating that the deck-level shock spectrum may significantly underestimate the shock loading of equipment mounted near the top of the cabinet.

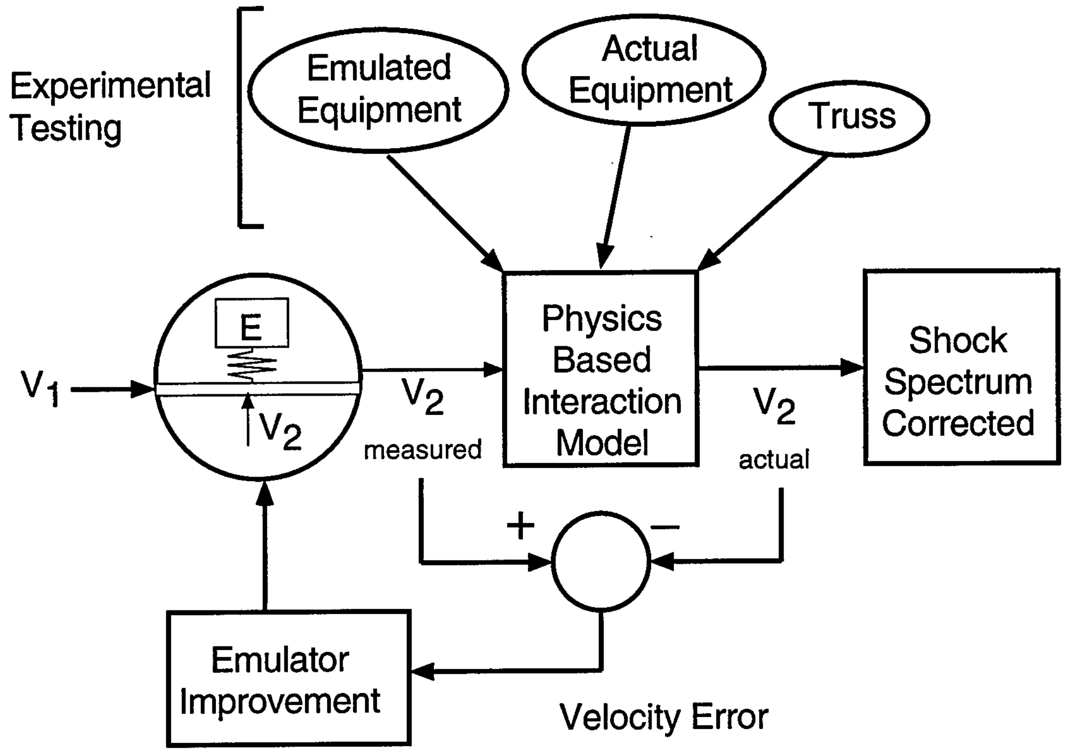

Figure 3 – Experimental procedure for shock trial analysis and emulator evaluation. Using mobility measurements obtained individually for the emulators, actual equipment, and truss, a deck-equipment interaction model is constructed. Using this model, deck velocities, v_2, to be measured during the ¼ scale shock trials, can be corrected for emulation error. In addition, the model motivates an analytical approach to emulator improvement.

List of Publications / Reports / Presentations

1. Papers Published in Refereed Journals

(i) ``Semi-active Control of Friction Dampers," P. Dupont, P. Kasturi and A. Stokes, *Journal of Sound and Vibration*, Vol. 202, No. 2, May 1, 1997, pp. 203-218.

(ii) ``Stability of Frictional Contact in Constrained Rigid-body Dynamics," P. Dupont and S. Yamajako, *IEEE Transactions on Robotics and Automation*, Vol. 13, No. 2, April 1997, pp. 230-236.

2. Refereed Conference Publications

(i) ``Periodic Optimal Control of Dampers," P. Kasturi and P. Dupont, 1997 ASME Design Engineering Technical Conference, Sacramento, CA, September 1997.

(ii) ``Experimental Identification of Kinematic Constraints," P. Dupont, T. Schulteis and R. Howe, *Proceedings of the 1997 IEEE International Conference on Robotics and Automation*, Albuquerque, NM, April 1997, pp. 2677-2682.

(iii) ``Automatic Identification of Remote Environments," T. Shulteis, P. Dupont, P. Millman and R. Howe, *Proceedings of the ASME Dynamic Systems and Control Division*, presented at the 1996 ASME International Mechanical Engineering Congress and Exposition, Atlanta, GA, November 1996, DSC-Vol. 58, pp. 451-458.

(iv) ``Experimental Evaluation of a Semi-active Friction Damper," P. Dupont, P. Kasturi and A. Stokes, *Elasto-impact and Friction in Dynamic Systems*, presented at the 1996 ASME International Mechanical Engineering Congress and Exposition, Atlanta, GA, November 1996, DE-Vol. 90, pp. 69-74.

3. Invited Presentations

(i) ``Stability of Rigid-body Dynamics with Sliding Frictional Contacts," University of Pennsylvania, GRASP Laboratory, June 27, 1997.

(ii) P. Dupont, "Equipment modeling for scale shock trials," Naval Surface Warfare Center, Carderock Division, March 1997.

(iii) ``Equipment Emulators for UNDEX Testing," ONR Program Review of Basic and Applied Research in Structural Dynamics and Structural Acoustics, Austin, TX, Feb. 18-21, 1997.

Qualitative and Quantitative Guidelines to Equipment Emulation in UNDEX Testing

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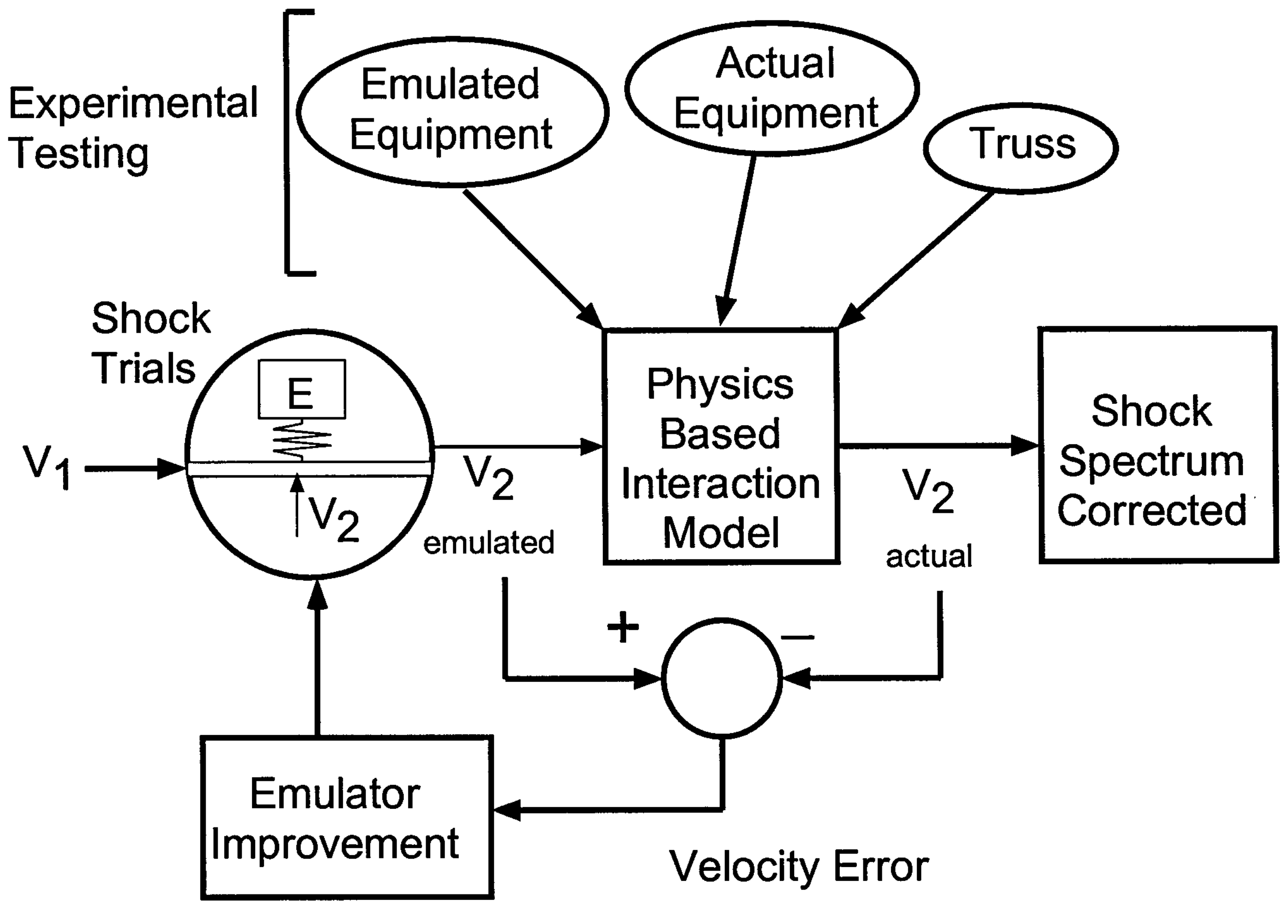
Shock Testing of Scale Models

- *Navy's goals:*
  - *Evaluate truss and mount designs regarding shock survival of COTS equipment.*
- *Our goals:*
  - *Provide tools for the design and analysis of scale shock experiments.*

Equipment Emulation Objectives

- *Develop general methods to:*
  - *Identify emulator dynamic properties that satisfy specified error and cost/complexity criteria.*
  - *Produce conceptual mechanical designs from the identified dynamic properties.*
- *Develop post-processing techniques to account for emulation errors so as to more accurately interpret shock trial data.*

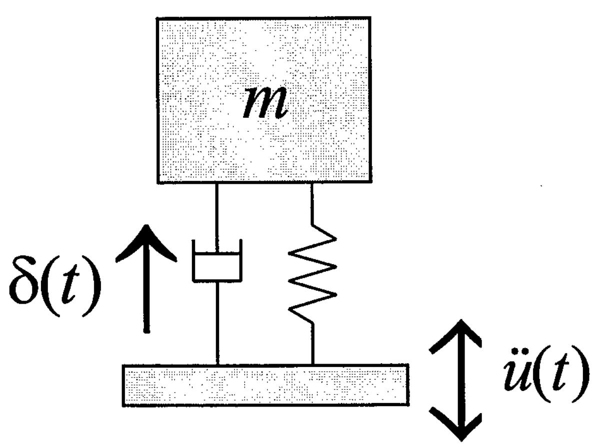
NSWCCD Shock Trial Design and Analysis

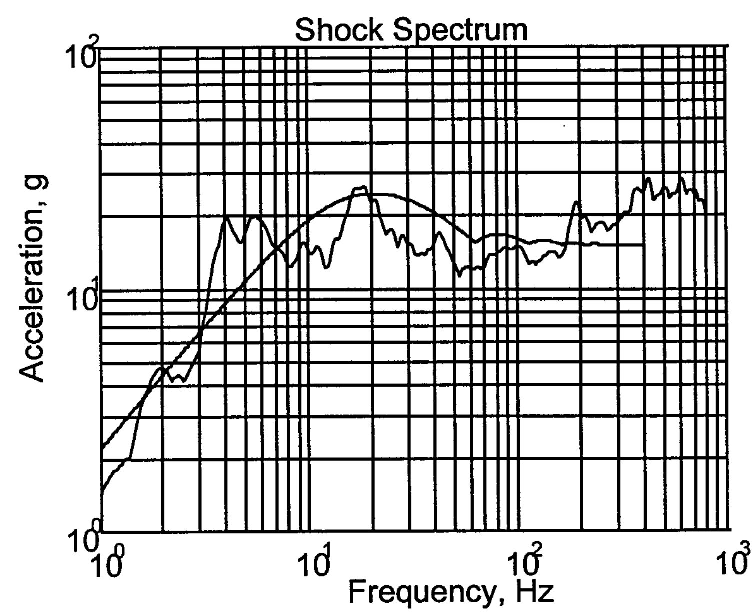


Describing Shock Events: Motion histories versus effect on structures

Shock spectrum: $$\delta(t) = \frac{1}{\omega_d}\int_0^t \ddot{u}(\tau) e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)\,d\tau$$

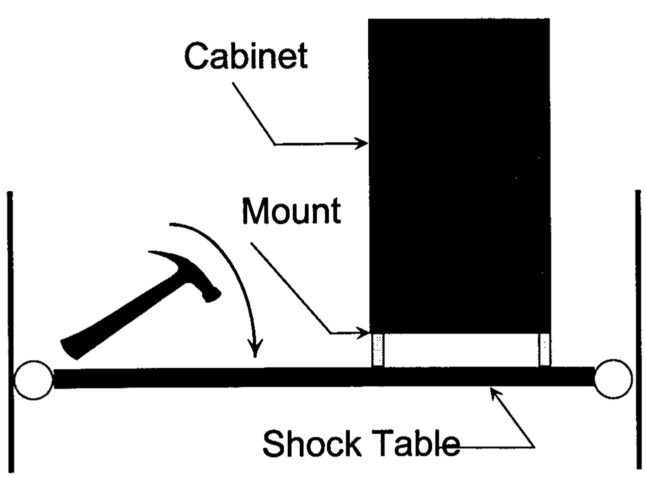
$$A(\omega_n,\zeta) = \frac{\omega_n^2}{g}\delta_{max}(\omega_n,\zeta)$$



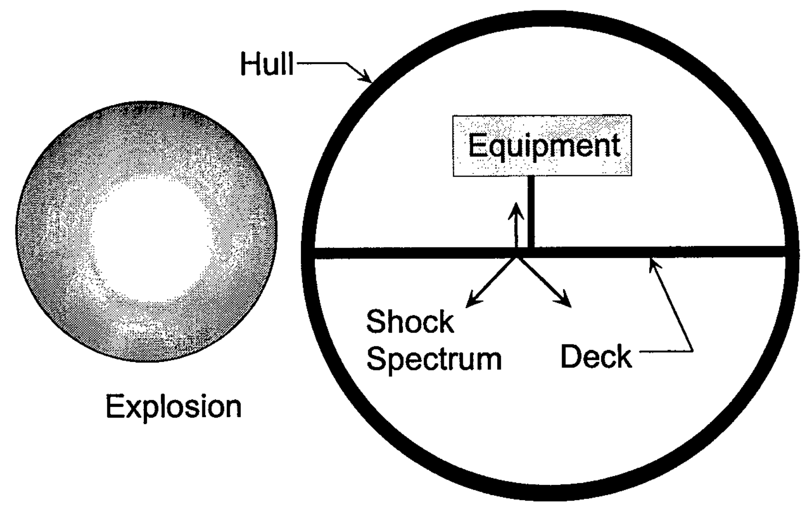


NAVSEA Deck and Equipment Qualification

- **Equipment:**
  - *Fully-loaded cabinets mounted on shock tables.*
  - *15g, 40 msec base motion applied in 3 directions.*
  - *If the equipment remains functional, it passes.*



- **Deck:**
  - *1/4 scale shock trials*
  - *15g half-sine acceleration pulse of 40 msec duration.*



Qualitative Guidelines

- *Shock spectrum versus Fourier spectrum:*

$$\Delta(\omega) = \mathrm{F}\{\delta(\mathrm{t})\} = G(\omega)\ddot{U}(\omega)$$

where $\ddot{U}(\omega)$ is the unit acceleration impulse response of the shock spectrum oscillator.

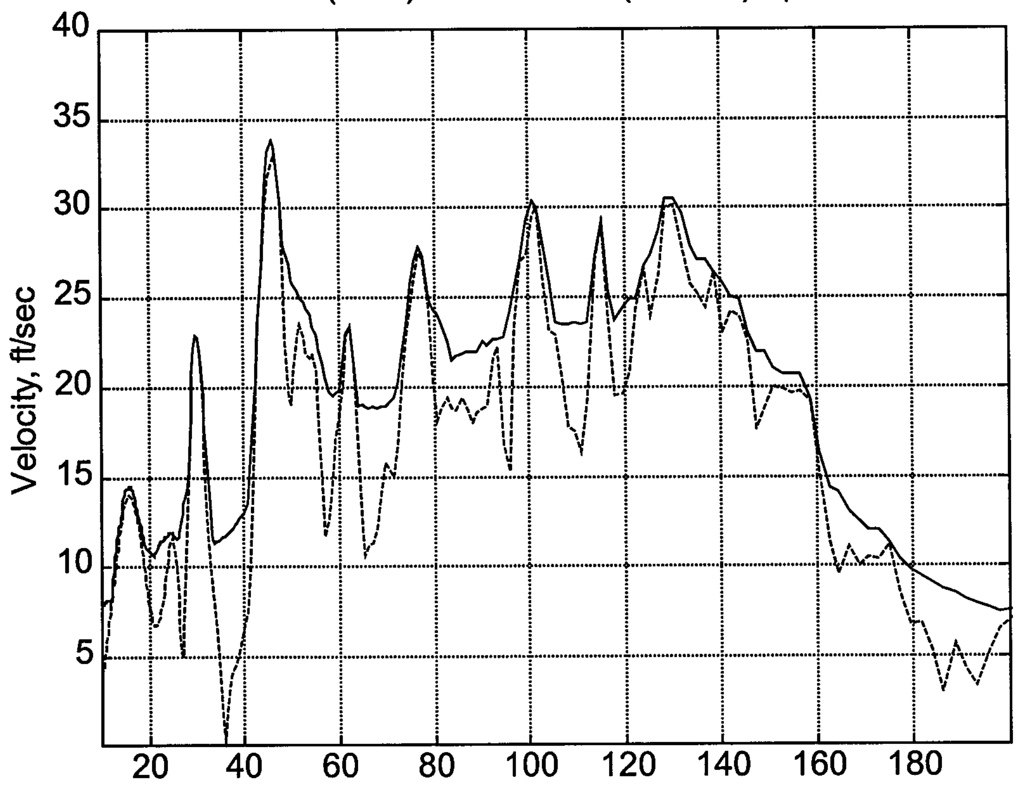
$$S(\omega) \geq S_r(\omega)$$

where $S_r(\omega)$ is the residual shock spectrum.

Undamped shock spectrum:

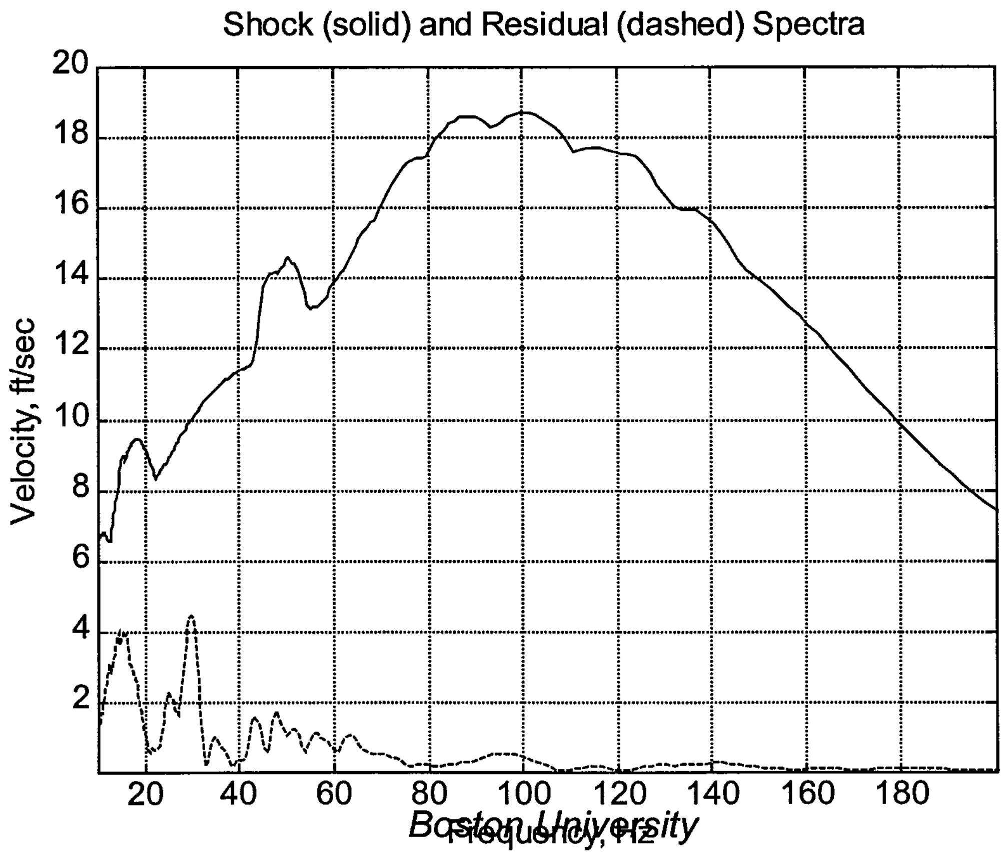
$$S_r(\omega) = \text{mag}\left\{\dot{U}(\omega)\right\} \qquad \text{maxima}\left\{\text{S}(\omega)\right\} \approx \text{maxima}\left\{S_r(\omega)\right\}$$

Shock (solid) and Residual (dashed) Spectra

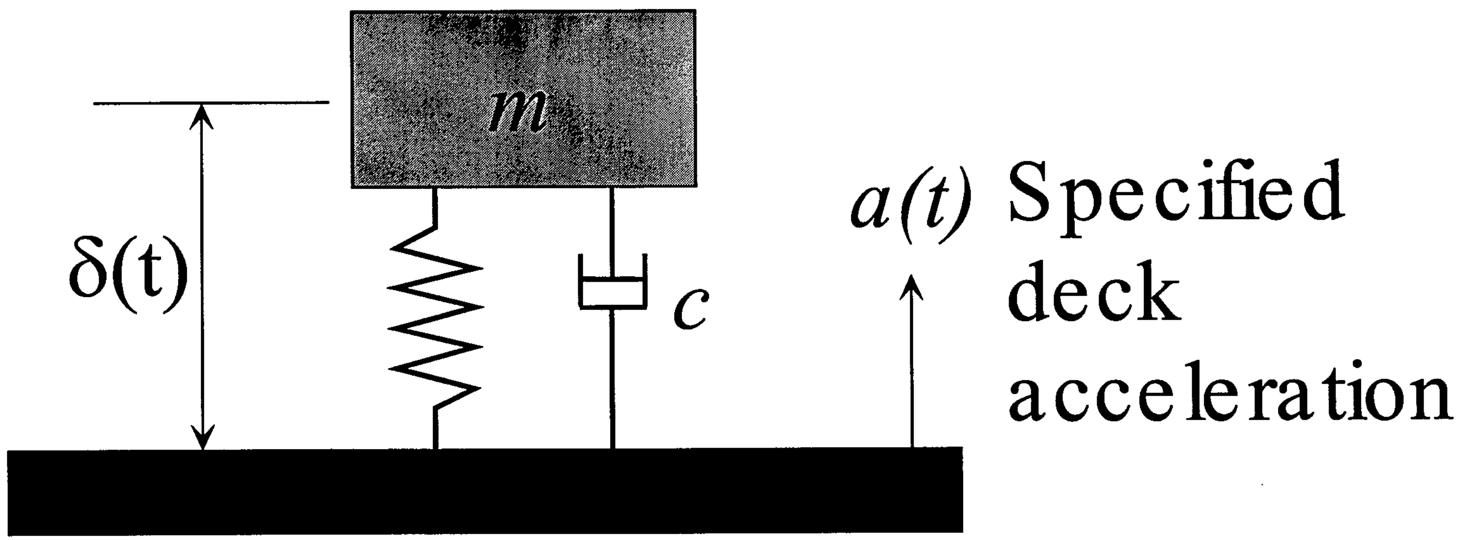


Damped shock spectrum (5%):

- $S(\omega) >> S_r(\omega)$
- Shock spectrum is infinity norm: $S(\omega) = \sup_t \left|\delta(t,\omega)\right|$



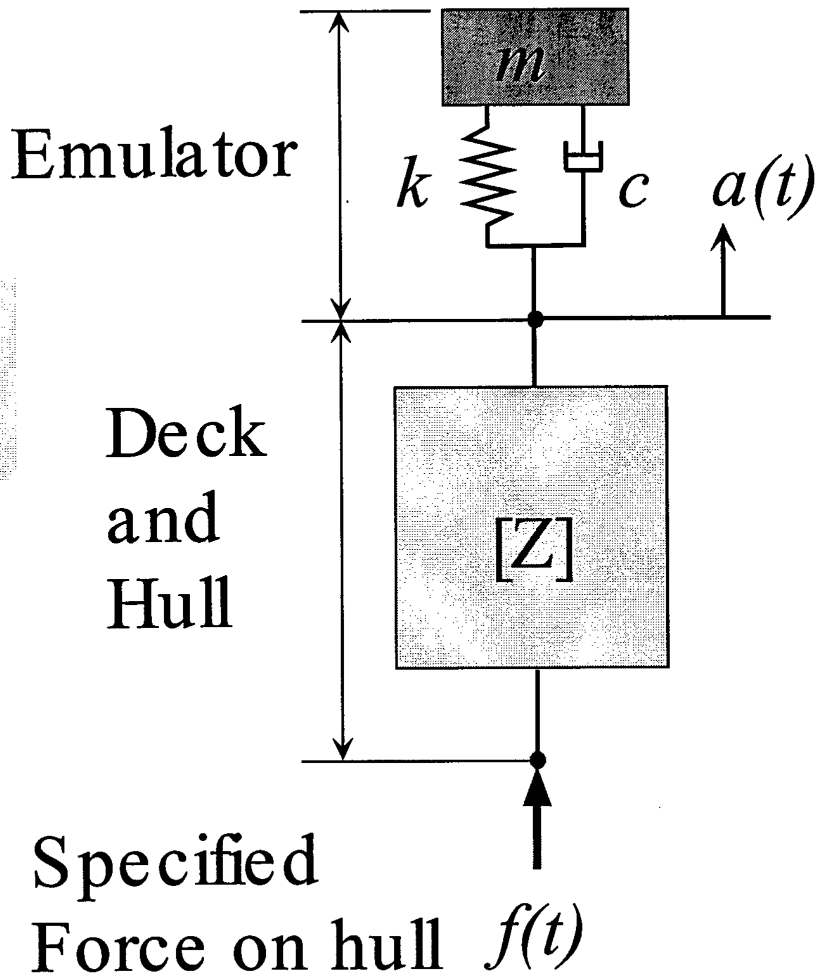
Damping in the Shock Spectrum



Increasing c always decreases $\delta(t)$

$$\delta(t) = \frac{1}{\omega_d}\int_0^t a(t-\tau)e^{-\zeta\omega_n\tau}\sin\omega_d(\tau)\,d\tau$$

Damping in the Emulator



- Increasing c in the emulator may increase or decrease $\delta(t)$ by increasing or decreasing $a(t)$.
- Classic example: damped vibration absorber.
- Specific numerical results (e.g. O'Hara and Cunniff) cannot be generalized.

Quantitative Guidelines

- How good is an emulator, i.e., how does its response compare with that of perfectly scaled equipment? ➔ *emulator error.*
- Is a particular emulator design change worth the cost, i.e., how sensitive is deck velocity to the change? ➔ *emulator sensitivity.*
- How can an emulator be built to meet specified error criteria? ➔ *emulator design.*

Emulator error

Naval practice:

- *compare attachment point impedance (or mobility) measurements of the emulator with that of the actual equipment.*

Our definition:

- *Emulator error is the difference between the deck velocity (at the equipment base) obtained with a particular emulator and that which would be obtained with the actual equipment. This may be also be expressed as a difference in shock spectra.*

Emulator Error

Z_{22} = deck attachment point impedance matrix

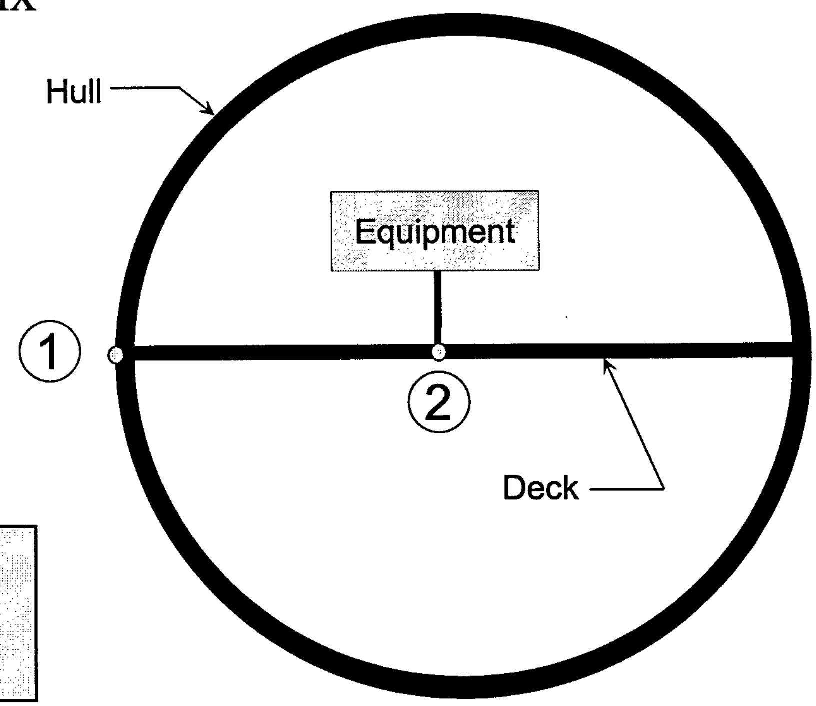
Z_e = emulator attachment point impedance matrix

Z_a = actual equipment impedance matrix

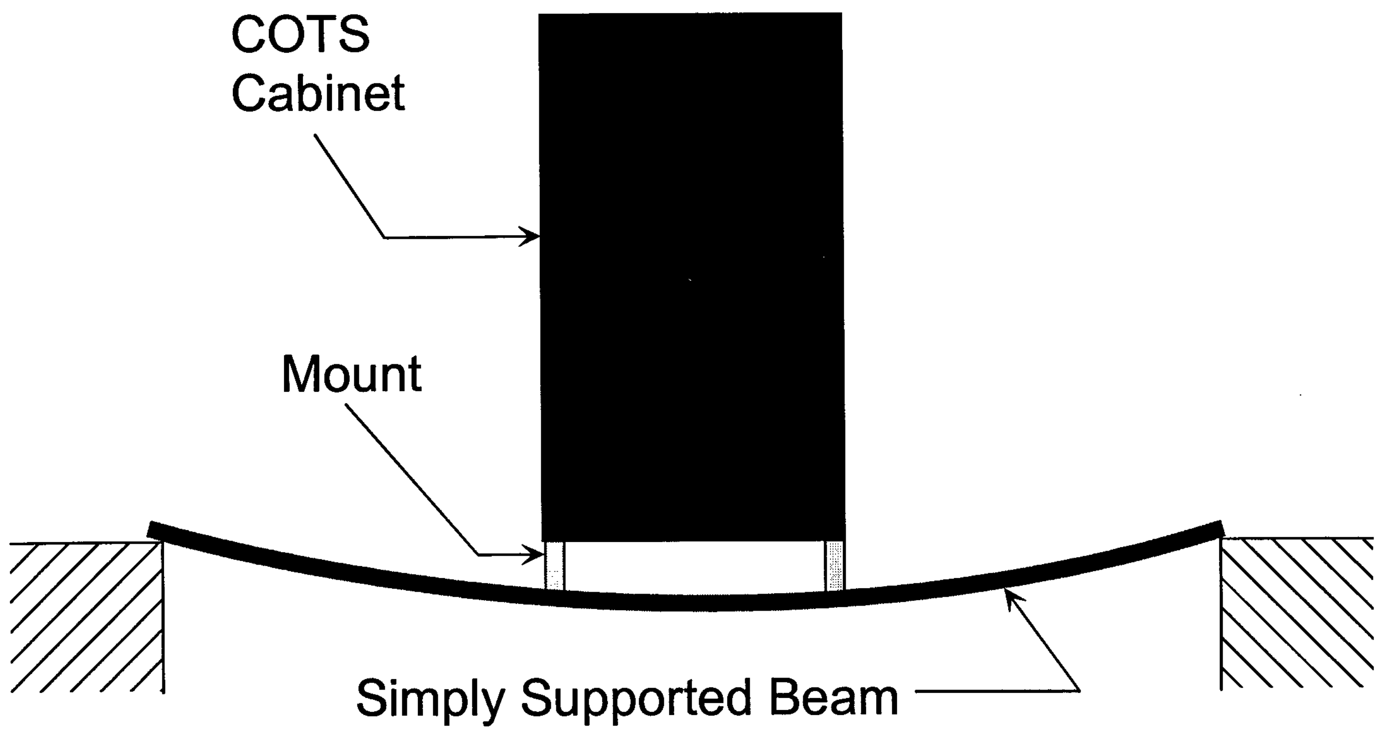
Deck + Hull:
$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Equipment:
$$F_2 = -Z_e V_2$$

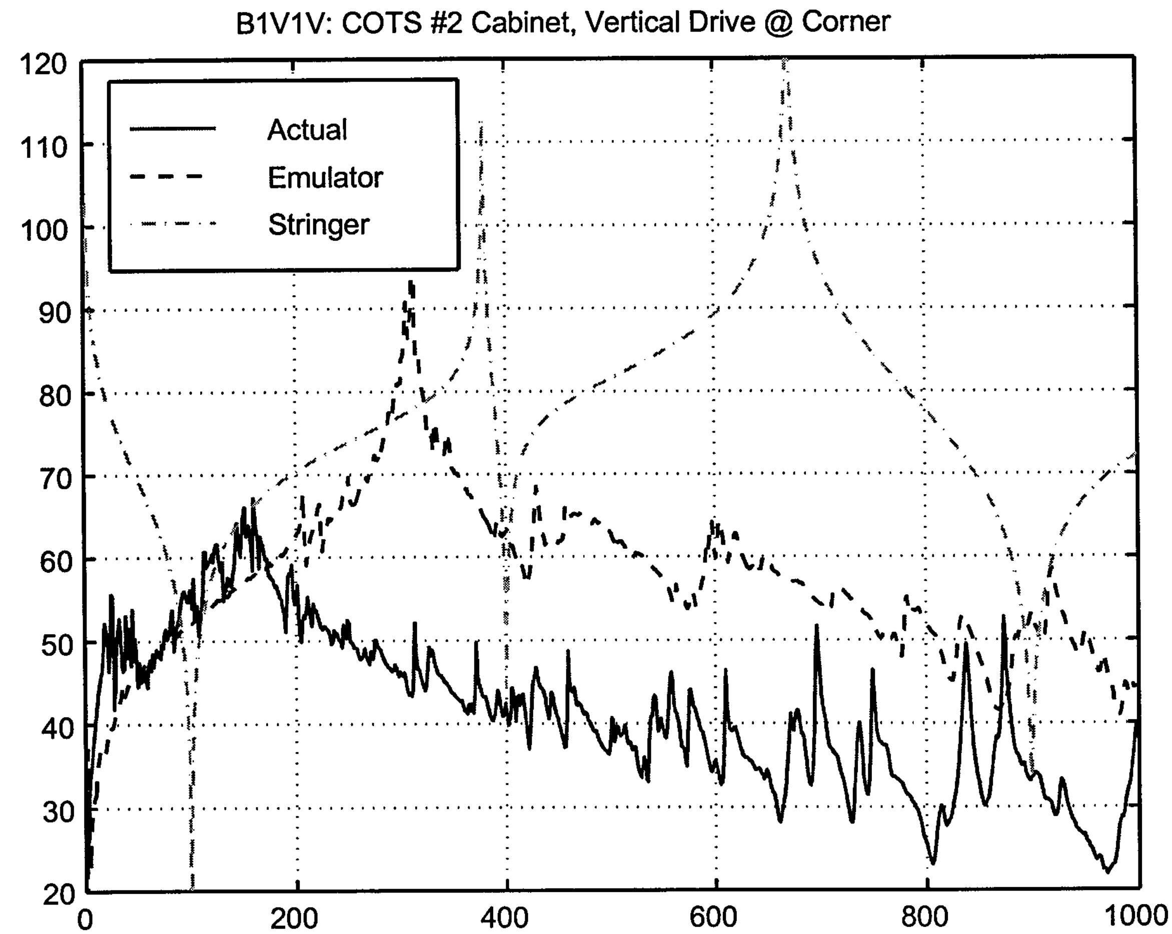
$$\boxed{V_{2a} = \left[Z_a + Z_{22}\right]^{-1}\left[Z_e + Z_{22}\right] V_{2e}}$$



Example: COTS cabinet on stringers



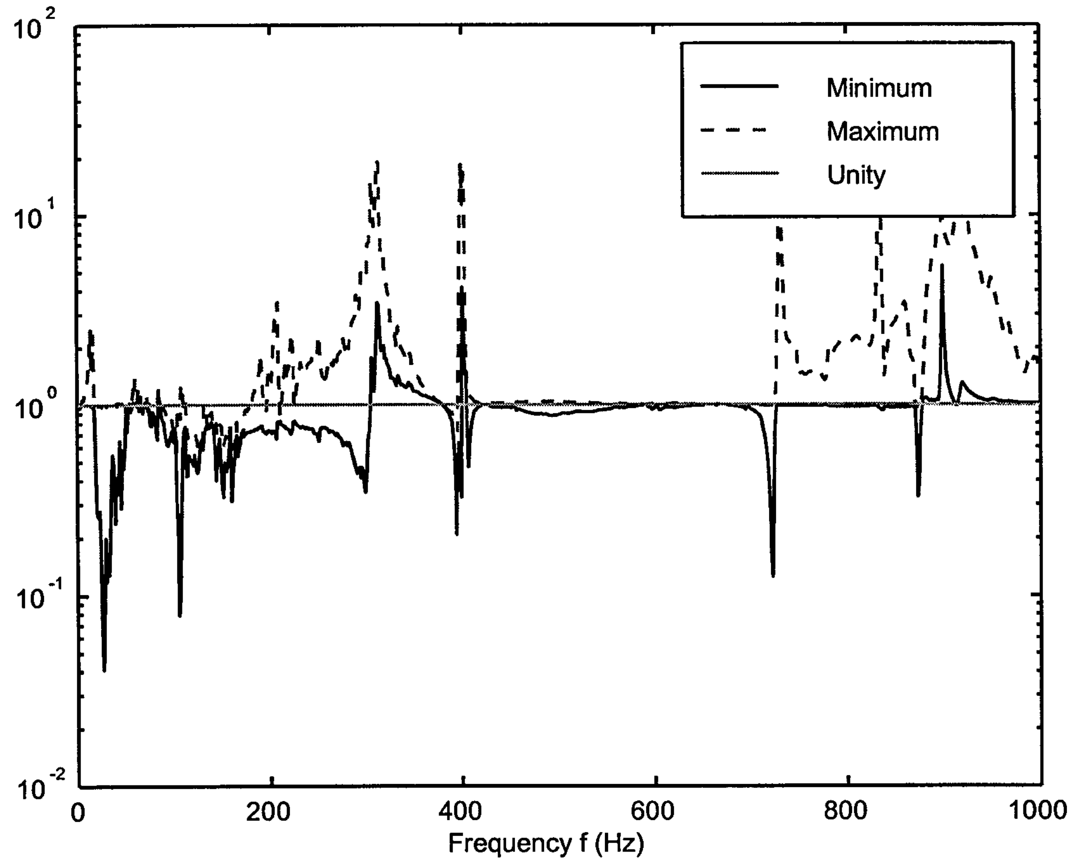
Naval practice: *Impedance of cabinets and stringer*



- *Ignores deck dynamics*
- *compares impedance magnitude*

New Approach: *Lower and upper bounds on actual deck velocity*

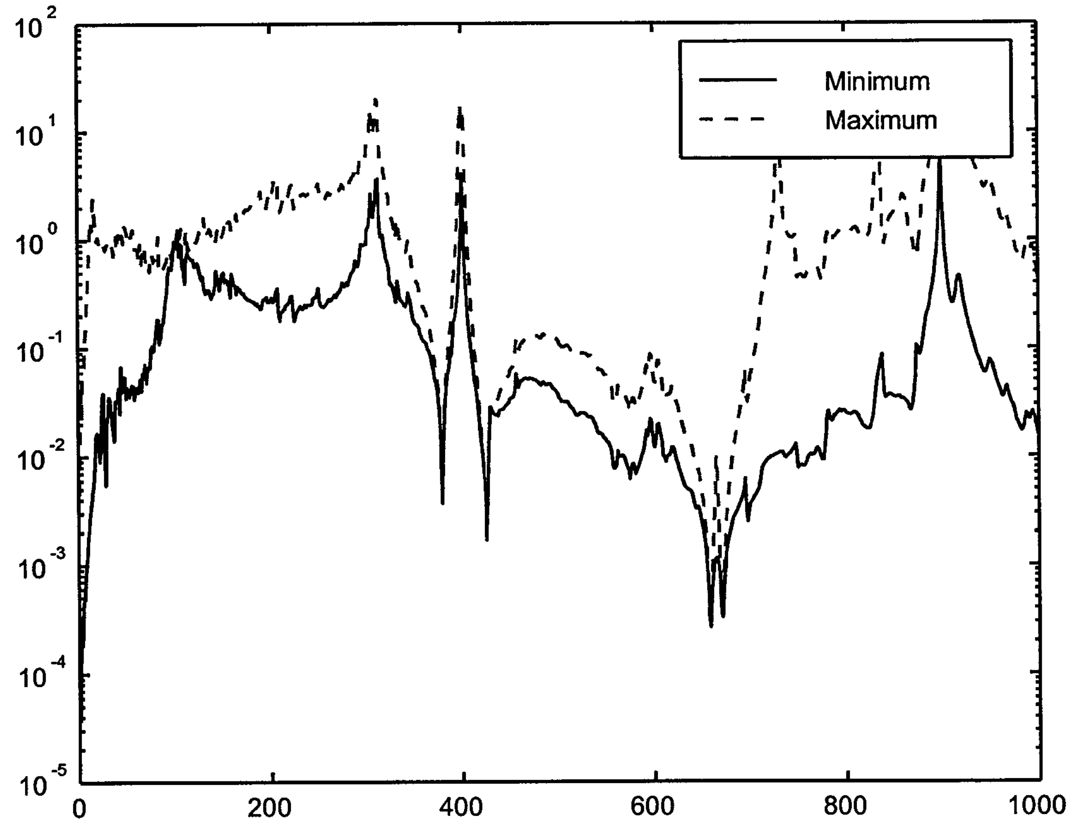
$$V_{2a} = \left[Z_a + Z_{22}\right]^{-1}\left[Z_e + Z_{22}\right]V_{2e}$$



- *Includes deck dynamics*
- *compares velocity magnitude*

New Approach: *Lower and upper bounds on velocity error*

$$E = V_{2a} - V_{2e} = \left(\left[Z_a + Z_{22}\right]^{-1}\left[Z_e + Z_{22}\right] - I\right)V_{2e}$$



- *Includes deck dynamics*
- *compares velocity magnitude and phase*

Shock spectrum versus velocity error

- Shock spectrum for velocity component i

oscillator displacement: $\delta_i(t) = \int_0^t v_{2i}(\tau) g(t-\tau) d\tau$

shock spectrum: $S_i(\omega) = \sup_{0 \le t < \infty} |\delta(t)|$

- Shock spectrum error

$$\Delta_{2e}(\omega) \longrightarrow \boxed{\begin{bmatrix} \ddots & & 0 \\ & G^{-1} & \\ 0 & & \ddots \end{bmatrix}} \xrightarrow{V_{2e}} \boxed{\begin{matrix} [Z_a + Z_{22}]^{-1} \\ \cdot [Z_e + Z_{22}] \end{matrix}} \xrightarrow{V_{2a}} \boxed{\begin{bmatrix} \ddots & & 0 \\ & G & \\ 0 & & \ddots \end{bmatrix}} \longrightarrow \Delta_{2a}(\omega)$$

New Result: *Bounds on shock spectrum error*

$$P(\omega) = \left[Z_a + Z_{22}\right]^{-1}\left[Z_e + Z_{22}\right]$$

- *Single shock spectrum:*

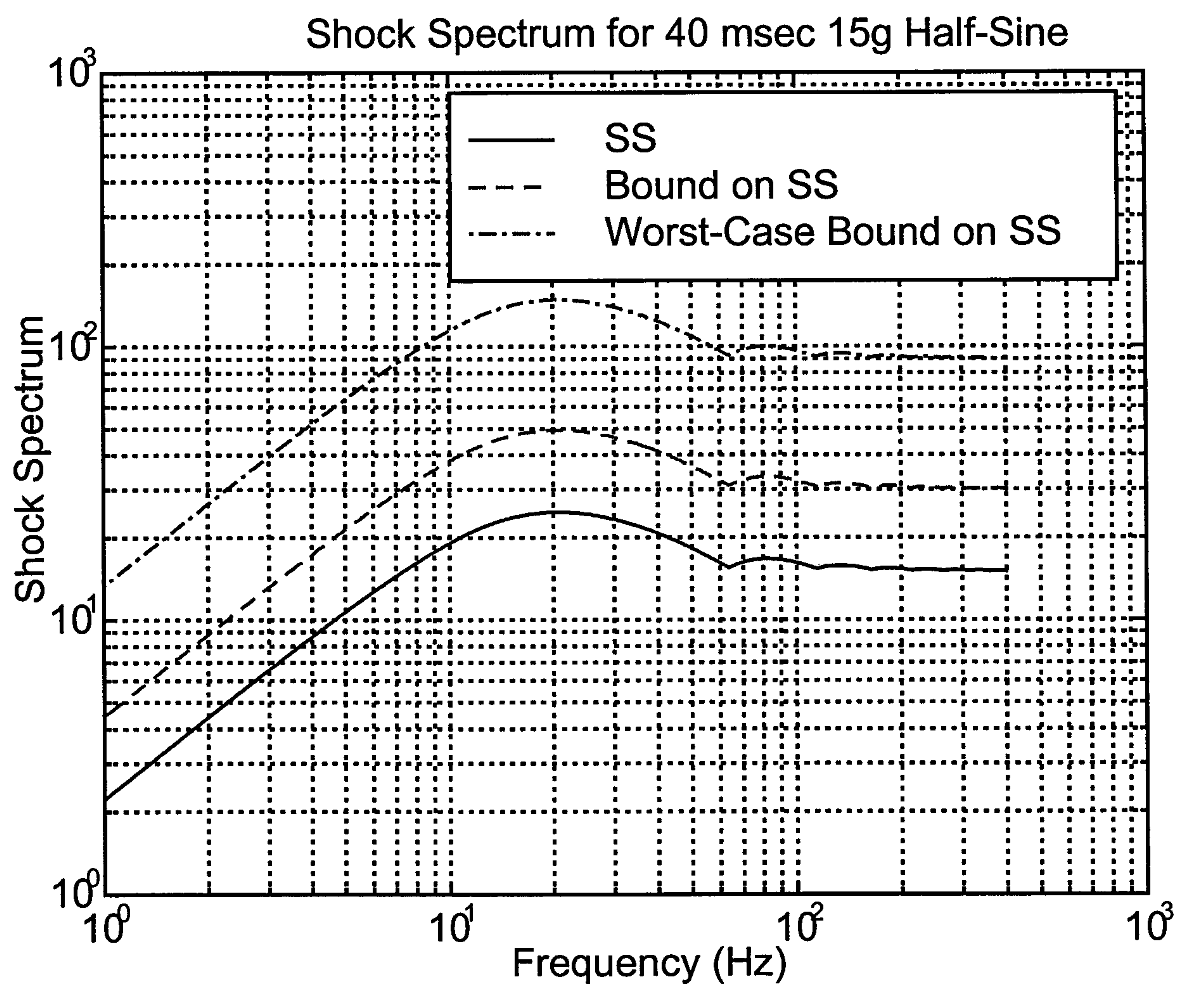
$$\left\|\delta_{ai}(t)\right\|_\infty = \sup_t \left|\delta_{ai}(t)\right| \le \sum_j \left\|p_{ij}(t)\right\|_1 \cdot \left\|\delta_{ej}(t)\right\|_\infty$$

  - *where* $\left\|p_{ij}(t)\right\|_1 = \int_0^\infty \left|p_{ij}(t)\right| dt$

- *Worst-case shock spectrum for emulator (or deck):*

$$\sup_t \max_i \left|\delta_{ai}(t)\right| \le \left(\max_i \sum_j \left\|p_{ij}(t)\right\|_1\right) \cdot \left(\sup_t \max_j \left|\delta_{ej}(t)\right|\right)$$

Anticipated single and worst-case shock spectrum bounds



Properties of shock spectrum bounds

- Multiplying factors for measured shock spectra to obtain worst-case actual shock spectra.
- May be greater or less than one. In case of zero error, bound is one.
- Independent of frequency.
- Useful error measure even without computing any shock spectra.
- Can be interpreted as a maximum percent error.

Quantitative Guidelines

- How good is an emulator, i.e., how does its response compare with that of perfectly scaled equipment? → *emulator error.*

- Is a particular emulator design change worth the cost, i.e., how sensitive is deck velocity to the change? → *emulator sensitivity.*

- How can an emulator be built to meet specified error criteria? → *emulator design.*

Emulator error sensitivity

- *Variations around the nominal system:*

$$Z_e = Z_e^n + \partial Z_e$$

$$E = E^n + \partial E$$

- *∂E is a complex vector representing the change in error due a change in emulator impedance, ∂Z_e*

$$\partial E = \left[Z_a + Z_{22}\right]^{-1} \partial Z_e V_{2e}$$

- *<u>Error sensitivity</u> is defined as the normalized change in deck velocity error due to ∂Z_e*

Quantitative Guidelines

- How good is an emulator, i.e., how does its response compare with that of perfectly scaled equipment? → *emulator error*.
- Is a particular emulator design change worth the cost, i.e., how sensitive is deck velocity to the change? → *emulator sensitivity.*
- How can an emulator be built to meet specified error criteria? → *emulator design.*

Emulator Design

- Given:
  - *set of major fixed-base modes with variable damping*
  - *parameterized model of additional modes (modal mass distribution, e.g., plate modes)*
  - *specified level of deck shock spectrum error*
- *Solve for the model parameters.*

Planned Navy interaction / Impact on Navy programs

- *Assist in planning, design and evaluation of 1/4 scale shock trials (FY '99):*
  - *Bill Martin, NSWCCD*
  - *Dr. Liming Salvino, NSWCCD*
- *Cabinet emulators*
  - *shock spectrum error prediction*
- *Structurally Integrated Enclosure emulators*
  - *design, modification and error prediction*

Summary

- *Developing new tools for assessing shock-trial emulator fidelity*
  - *evaluate existing designs - error*
  - *predict value of design modification - sensitivity*
  - *formalizing a design procedure*
- *Expressed in NAVSEA's Coin of the realm*
  - *deck velocity and shock spectra*
- *Using easily obtained data*

16th International Congress on Acoustics and 135th Meeting Acoustical Society of America, Seattle, 20-26 June 1998.

AN ERROR MEASURE FOR THE SHOCK TESTING OF SCALE MODELS

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Abstract: In a variety of situations, an undesired shock excitation is applied to a master structure that supports shock-sensitive equipment. Often, one wishes to design and test a master structure that transmits the least amount of shock energy to the attached equipment. In scaled testing of new designs, a major task is to design and construct "equipment emulators" – inexpensive mechanical systems which approximately mimic the dynamic behavior of the actual full-scale equipment as seen by the master structure. A new method is presented for assessing the fidelity of equipment emulators and for interpreting test data taken in the presence of imperfect emulators. The proposed approach uses easily obtainable frequency-domain impedance descriptions of the master structure and actual equipment at the attachment points. These ideas may provide a path by which experimentalists can efficiently arrive at conceptual designs of emulators that promise a specified degree of fidelity in terms of attachment point velocities and their associated shock spectra. The ideas are illustrated by application to the emulation of commercial-grade electronic cabinets for the testing of novel ship deck structures.

INTRODUCTION

In the design and use of scale equipment models for shock testing, there are two principal objectives. The first is to produce conceptual mechanical designs that satisfy specified error and cost/complexity criteria. Given a mechanical emulator, the second objective is to develop post-processing techniques that account for emulation error in the interpretation of shock trial data.

Prior work on the design of mechanical equipment emulators is limited to acoustic performance. The design approach consisted of reproducing the first four fixed-base equipment modal frequencies and masses. Design refinement involved adding damping materials to the nominal design so as to minimize the difference between drive-point impedance of the actual and scaled equipment at the attachment-points (1). With regard to shock, Barbone is developing numerical equipment models, described by a small number of physically motivated parameters, that reproduce early-time relations between forces and displacements at the attachment points (2).

In both approaches, a physical understanding is employed to obtain a simplified model of an otherwise highly complex dynamic system. The modeling is performed independent of the dynamics of the master structure. And while the latter approach directly addresses error criteria during modeling, neither provides a means to post-process experimental data to account for emulation error. A significant issue is the lack of a generally accepted definition for emulation error in the context of shock loading and an easily evaluated metric for assessing this error. The contribution of this paper is to propose such a measure.

EMULATOR ERROR

Emulator error is evaluated in the context of the scaled master structure. It is defined as the vector difference between attachment-point velocities obtained with a particular emulator and those that would be obtained with perfectly scaled equipment. It can be expressed as a transfer function matrix relating measured scale model velocities to the velocity error vector. As a metric of emulation error, the maximum and minimum singular values of the transfer function matrix can be plotted as a function of frequency. These values represent the maximum and minimum gains for all possible attachment-point velocity vectors. By taking appropriate norms of a related transfer function, emulation error can also be expressed in terms of shock spectrum bounds. Using the proposed transfer functions, experimental data can be corrected for emulator error. Using norms, experimental shock spectrum error can be bounded.

As a simple illustration of this method, Figure 1 depicts an equipment cabinet mounted on a master structure consisting of a simply supported beam. Considering only vertical motion, an analytical model of the beam, together with experimental data from actual and scale model cabinets, is employed. Comparison between drive-point impedance of the actual and emulated cabinets in Figure 2 suggests minimal error from 0-200 Hz and large errors outside that range. In contrast, the singular values of the 2×2 velocity error transfer function matrix shown in

Figure 3 indicate significant error from 0-400 Hz and small error from 420-700 Hz. Note also the effect of master structure impedance on error. Roughly speaking, a modest error is amplified (attenuated) when stringer impedance is small (large). The proposed method offers the advantages of: (i) ease of application, (ii) error evaluation in the context of the master structure, and (iii) a means for considering all possible attachment-point velocities.

ACKNOWLEDGEMENTS

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2. Cherukuri, A. and Barbone, P., "High Modal Density Approximations for Equipment in the Time Domain," *Journal of the Acoustical Society of America*, in press, 1998.

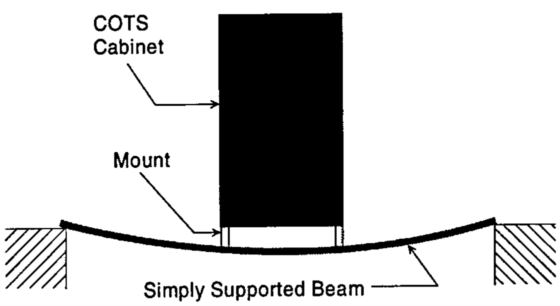

FIGURE 1. Equipment cabinet mounted on beam.

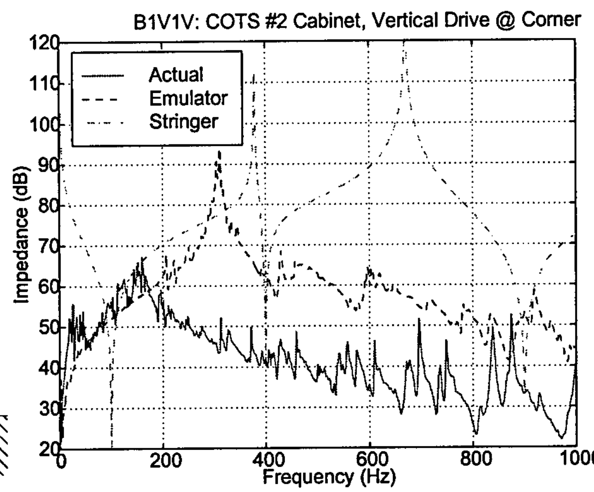

FIGURE 2. Vertical drive point impedance of cabinet, emulator and stringer (beam).

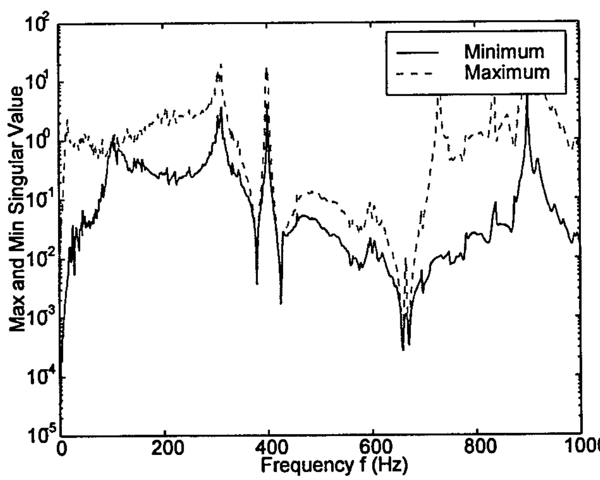

FIGURE 3. Maximum and minimum singular values of transfer function matrix relating measured vertical attachment point velocity to velocity error.